Provide Motivation Through Exciting Materials in Mathematics and Science

Sample Units

GB

PROVIDE MOTIVATION THROUGH EXCITING MATERIALS IN MATHEMATICS AND SCIENCE

Sample Units

English version

2014

2nd edition

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This work is available electronically on the project web page:

http://www.msc4all-project.eu/

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FOREWORD to 1st edition

Project PROMOTE MSc – Provide Motivation Through Exciting Materials in Mathematics and Science is a project under the COMENIUS 2.1 programme of the European Commission.

The aim of this project is to address the problem of the shortage of young people attracted to study and enter teacher training in the mathematical and scientific subjects of the school curriculum. We want to produce and gather materials in a European collaboration which intends to motivate students and teachers to be more interested in learning mathematics and science. Materials produced will be used by lecturers with their students at training institutions and by students on practice in schools. The material will be evaluated and disseminated through the established European network.

This booklet contains all the unit descriptors for the materials collected by the project. A unit descriptor briefly lists the unit title, aims, content and includes brief notes on resources and other matters so a teacher can decide on use of the unit.

Project Team

The project participants are teacher training institutions in four European Countries: The University of Sunderland (United Kingdom), the University of Vienna (Austria), the Palacky University Olomouc (Czech Republic), and the Constantine the Philosopher University Nitra (Slovakia).

Andreas Ulovec (AT)
Coordinator

Partners:

Soňa Čeretková (SK)
Neil Hutton (UK)
Josef Molnár (CZ)

FOREWORD to 2nd edition

After almost 10 years, it was time to revisit the materials, to use the numerous feedbacks that we received from teachers, and to improve the materials. For this reason we planned the project "MSc4All – Motivating Methods and Materials in Maths and Science: Dissemination" in the framework of the Lifelong Learning Programme, which allowed the project team to collect suggestions for improvements and put them into practice, as well as to produce and disseminate a second edition of the project materials. By this, we hope to come even closer to our original goal to increase the motivation to learn mathematics and science. The second editions of the project materials can be found at the webpage of the project:
http://www.msc4all-project.eu/

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MATHEMATICS

Lifelong
Learning
Programme

Unit Title	**Just a fraction more**
Topic	Arithmetic
Name and email address of person submitting unit	Andreas Ulovec Andreas.Ulovec@univie.ac.at
Aims of unit	To make students familiar with the notion of fractions and allow them to be able to compare and operate with fractions.
Indicative Content	Game with fractions.
Resources needed	None.
Teachers notes	None.

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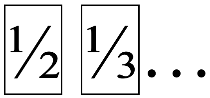
Just a fraction more

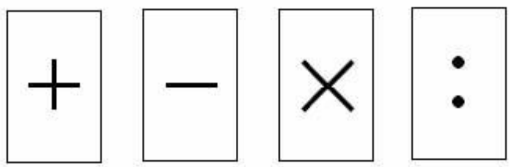
We want to describe a game approach for comparing fractions and for using addition, subtraction, multiplication and division of fractions.

Game 1: You will need: Sheet of paper, pen, pack of cards "fractions" showing all fractions that can be built with nominators "1,2,3,4,5" and denominators "1,2,3,4,5":



The cards are shuffled at the beginning of the game. Each player gets 5 cards, the rest remain as a spare stack – face down – where both players can reach it. The younger player begins by playing one of his or her cards. The other player tries to answer with a higher card. If he can do that he gets both cards and puts them on his stack. If he answers with a lower card, the first player gets them both and puts them on his stack. If he answers with a card of the same value, the first player has to play another card, the second player has to answer again. In this case, the player with the higher card gets all of the cards displayed. After each round both players take as many cards from the spare stack as is necessary to have 5 cards in his or her hand again (If the spare stack is gone, the players play until all their cards are gone). Then the roles are changed and the second player begins. At the end, each player counts the number of cards on their respective stack. These are the points won by the player in this game. The game is continued until one of the players reaches 100 points.

Game 2: You will need: Sheet of paper, pack of cards "fractions" (see above), pack of cards "operators" showing "+" (eight cards), "-" (six cards), "x" (five cards), ":" (three cards):



Again the “fraction” cards are shuffled, each player gets five of them, the rest remain as a spare stack. The “operator” cards are shuffled as well and put face down in the middle of the table. The younger player begins. He chooses two of his cards (we call them card A1 and B1) and puts them on the table. The other player chooses two cards as well and puts them on the table (card A2 and B2). Now both players draw one card each from the “operator” stack (we call them op1 and op2) and look at them (but do not show them to the other player). The first player has to decide now whether the result of his operation will be higher or lower than the result of the other player’s operation and he announces his guess aloud. He then puts his operation card on the table and decides the sequence: A1 op1 B1, or B1 op1 A1. Then the second player puts his operation card on the table and decides about the sequence as well. If the first player was correct with his guess, he gets one point. If the guess was wrong, the second player gets one point. Both players put the two fractions cards used in this round aside and draw new fractions cards from the spare stack so that they have again 5 cards in their hand. Also, the operations cards used are put aside. The following rounds have the same rules, only the player who begins changes in every round. The winner is the player with the most points.

Lifelong Learning Programme

Unit Title	**The smarter, the faster!**
Topic	Vectors
Name and email address of person submitting unit	Silke Fürweger fuersilke@yahoo.de
Aims of unit	The unit describes a “trivial pursuit”-like game to enable students to become familiar with addition, subtraction and multiplication of vectors
Indicative Content	Algebra
Resources needed	None.
Teachers notes	None.

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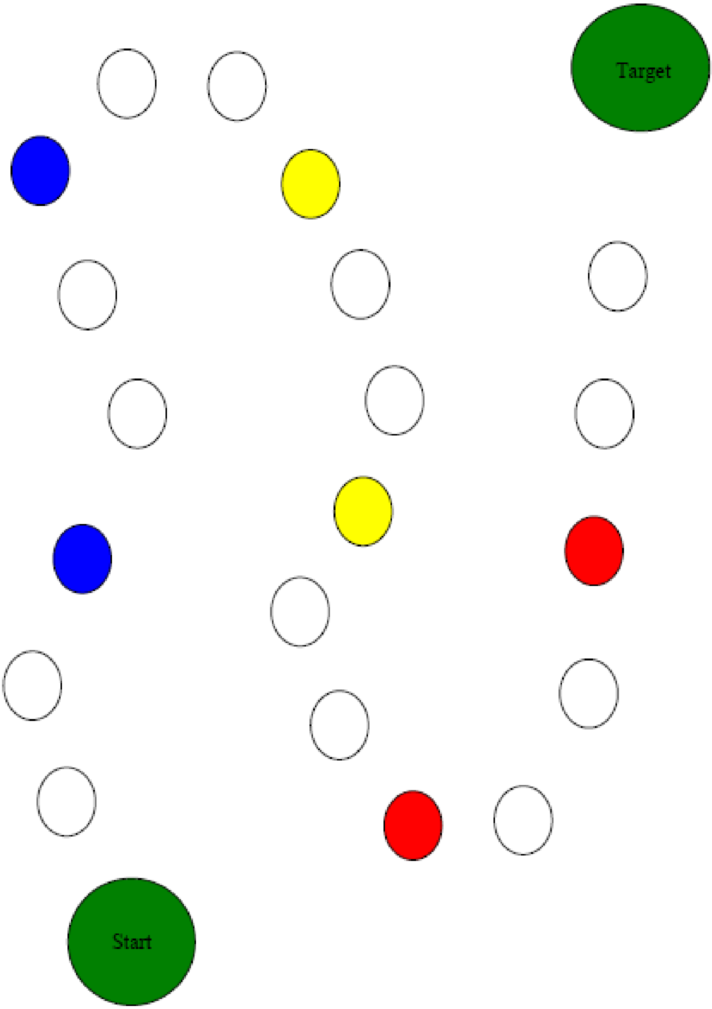
The smarter, the faster!

You need: 2-6 players, game board, a corresponding number of counters, number cards (36 cards, 12 with number “1” written on one side, 12 with “2”, 12 with “3”), action cards in the colours blue, yellow and red, solution cards in the same colours, paper and pencil.

The aim of the game is to get your counter as fast as possible to the target field. At the beginning, both counters are positioned at the “start” field. Instead of a dice, you will have number cards with the numbers “1”, “2” or “3” written on them. The cards are shuffled with the number side facing down. Each player draws one card. The player with the highest number begins. If cards are equal, the players with the highest numbers draw again.

One move consists of two parts. First, the player draws a number card and moves the corresponding number of fields. Second, if he or she moves over or ends on a coloured field, the player has to draw an action card and try to solve the problem on it. If the player is able to successfully solve the problem (if in doubt, see the corresponding answer card), he/she is allowed to move to the field succeeding the next field of the same colour. If he or she is not able to solve the problem, the player has to remain on the coloured field and draw another action card in the next round. Should the player be unsuccessful at one colour three times in a row, he or she has to move the counter back to the “start” field. After the move ends, the next player draws a number card and so on. The winner is the player who reaches the target field first. If the number cards are used up, just shuffle them again.

Game board:



Action cards: You will need about 3 cards per player per colour. Each colour marks a particular kind of problem:

- Blue – Word problems concerned with addition or subtraction of vectors
- Yellow – Word problems concerned with multiplication of a vector with a scalar
- Red – Numeric/algebraic addition, subtraction or multiplication problems.

Such problems can easily be constructed or taken from a text book or the internet. Below is an example of each kind of problem.

- Blue: Four grocery stores get their vegetables from two farmers. Vector A = (30, 17, 56, 29) describes how many tons of vegetables each of the four stores get from the first farmer each year, Vector B = (45, 23, 54, 70) describes the same for the second farmer. Vector C describes how many tons of vegetables the four stores get from both farmers together. Express C by means of A and B and calculate C. Vector D describes how many tons more the stores get from the second farmer compared with the first farmer. Express D by means of A and B and calculate C.
- Yellow: The prices (in Euros) for five products are given by the vector V = (21, 39, 45, 79, 54). If one buys more then 100 pieces each one gets 15% discount. Express the vector V' that describes the discount prices of the products. What is the relationship between V and V'?
- Red: A = (–4, 3, 0), B = (6, –2, 5). Subtract the double of A from the quadruple of B.

Lifelong Learning Programme

Unit Title	**Exercises for promoting space imagination**
Topic	Geometry – 3D geometry
Name and e-mail address of person submitting unit	Josef Molnár josef.molnar@upol.cz
Aim of unit	To promote space imagination in solid geometry using some exercises of different level.
Indicative Content	3 lessons Age of students: 14+
Resources needed	Drawing tools, cardboard, glue, scissors, wire, models of solids.
Teachers notes	Work in groups.

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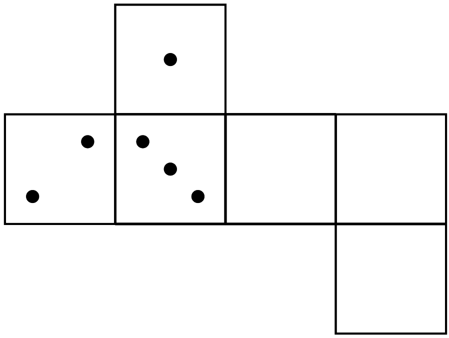
Exercises for promoting space imagination

Problems:

1. Complete the net of a die with points so that there are seven of them on each two opposite faces.

a)



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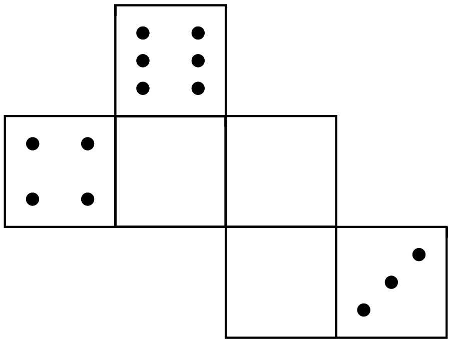
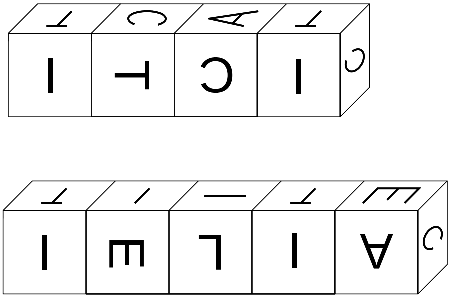
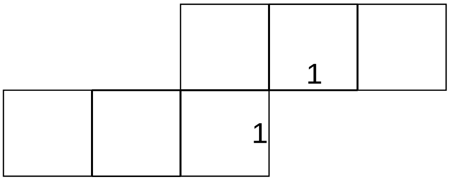


2. Find and sketch as many nets of a cube as you can. There are 11 of them. (Two nets are considered congruent if you can transform one onto the other so that they coincide.)

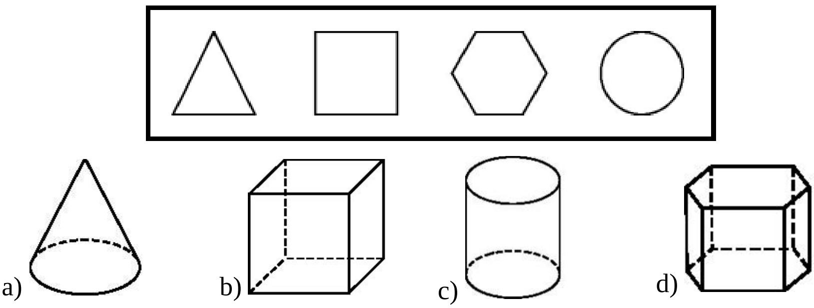
3. A writing (the first two words of a famous quotation) is made of nine equal cubes. You can see it from the reverse side. Find the words. Do you know which famous quotation it is?



4. In the given net of a cube, mark by the same number the sides of squares which form the same edge of the cube (see the figure). Try also to do that in other nets of a cube, as well as in the nets of other solids.

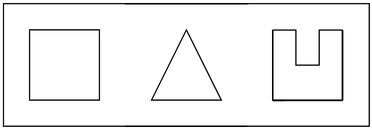


5. Match each of the given solids to all the “holes” through which it can be tightly (without gaps) pushed to the other side. (It becomes a bung at one moment.)



6. In oblique projection, draw a solid which can be tightly (without gaps) pushed through all three given “holes”.

a)



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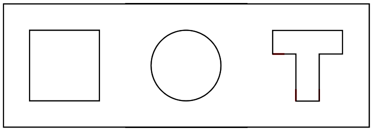
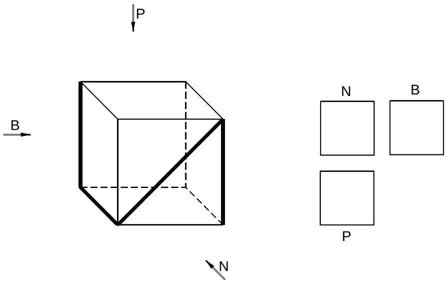


7. Construct the front view N, the top view P and the lateral view B of the wire drawn in oblique projection.



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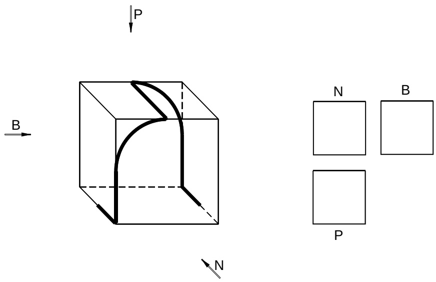
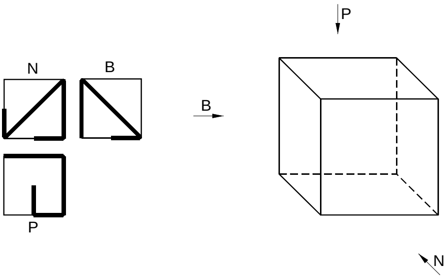


b)

8. Draw the wire (it does not tee) into the cube given in oblique projection using its front view, top view and lateral view.



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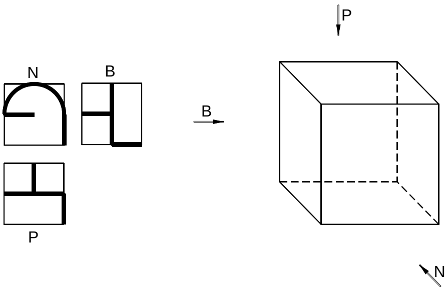
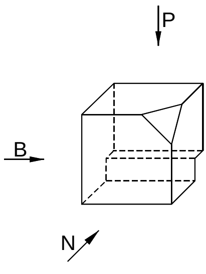


b)

9. Construct the front view, the top view and the lateral view of the solid in oblique projection.



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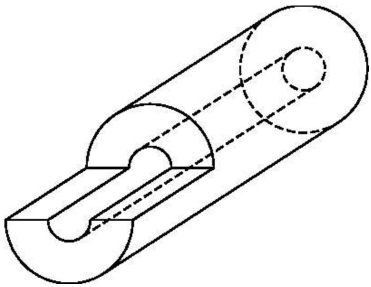
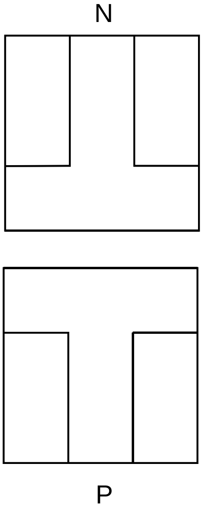
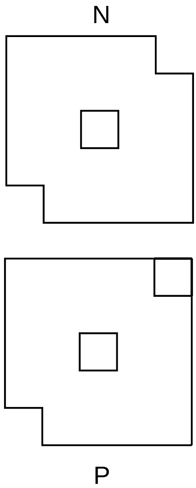


b)

10. Sketch the lateral view and the oblique projection of the solid using its front and lateral views. (There are more solutions.)

a) 

b) 

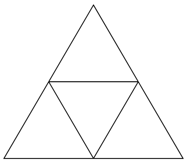
11. What is the shape of the perpendicular projection of:

a) a regular tetrahedron whose two edges are parallel to the plane of projection?

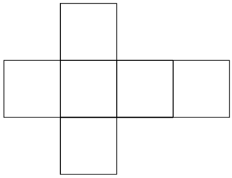
b) a cube whose solid diagonal is perpendicular to the plane of projection?

12. Add the folds and make your own models of all five regular polyhedral (Platonic solids).

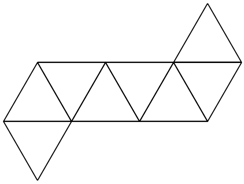
a) tetrahedron



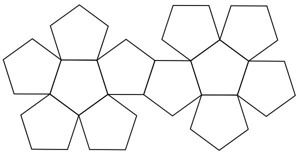
b) cube



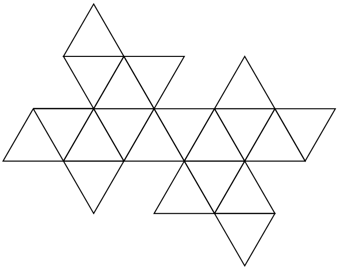
c) octahedron



d) dodecahedron



e) icosahedron



13. Given a triangular pyramid *ABCV* with the vertex *V*. The plane ρ intersects its edges *AB*, *BC*, *CV*, and does not pass through any of its vertices. Which other edges of the pyramid does the plane intersect?

14. Is it possible to construct a section of a cube which is:

a) an equilateral triangle,

b) an isosceles triangle ,

c) a scalene triangle,

d) an acute-angled triangle,

e) a right-angled triangle

f) an obtuse- angled triangle,

g) a square,

h) a rectangle,

i) a rhombus,

j) a trapezium,

k) a pentagon,

m) a hexagon,

n) a regular hexagon?

15. Given a regular tetrahedron *ABCD*. Points *P, Q, L, K* are the mid-points of the edges *AD, BD, CB, CD*, respectively. Find the angle between the lines *PQ* and *KL*.

16. Show that if we “go” on the edges of a cube (and dodecahedron), we can make an unbroken line through all vertices so that there is no double line on any of the edges. Try it also for other solids.

17. We will colour the faces of a cube white or black. They can be all white, all black or some white and some black. How many different cubes can we make?

18. How many unit cubes (= cubes with the edge length 1 unit) can be intersected by the solid diagonal of a cuboid with the edge lengths 5, 4 and 3?

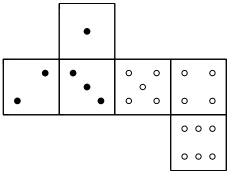
19. How many planes of symmetry are there in each Platonic solid?

20. Six different planes intersect a regular tetrahedron. Each of them passes through one edge of the tetrahedron and the mid-point of the opposite edge. How many solids will we get provided that we make all six sections at the same time?

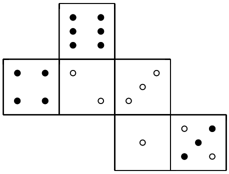
21. Given six different planes and a line p. The given line p is a part of three of the given planes. Two of the given planes are parallel, and intersect the line p. How many lines of intersection are there?

Answers:

1.



a)



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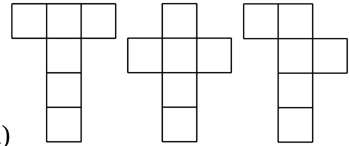

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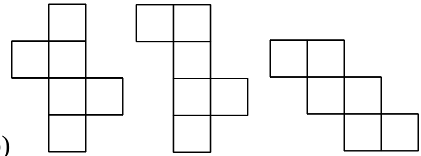

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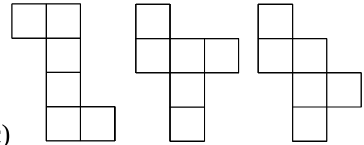
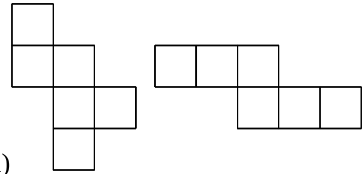
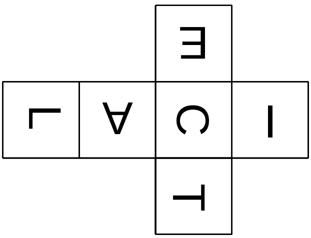


c)



d)

3. ALEA IACTA



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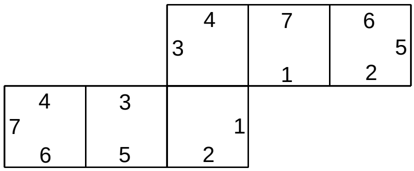
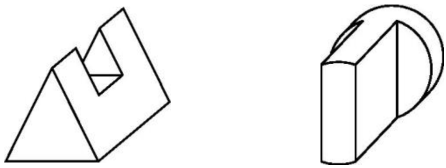


5. a-1, a-4, b-2, c-2, c-4, d-3

6.



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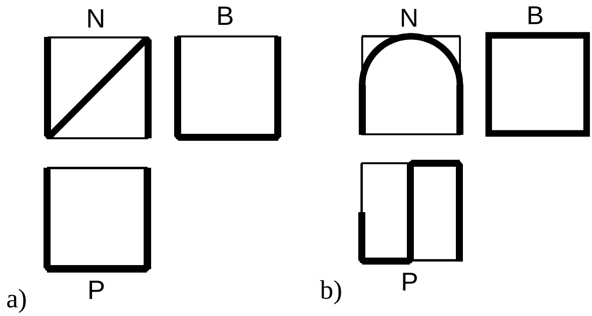

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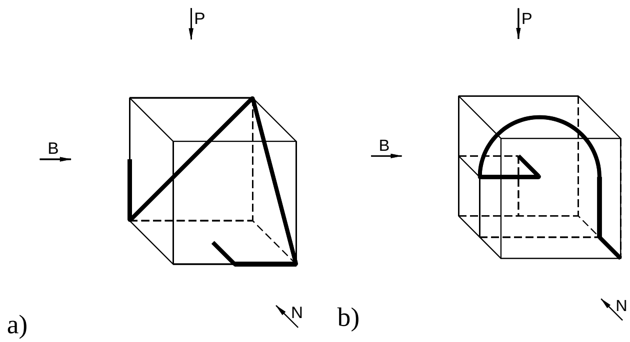

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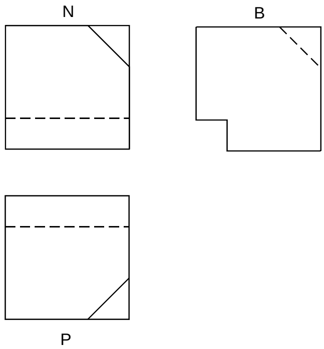

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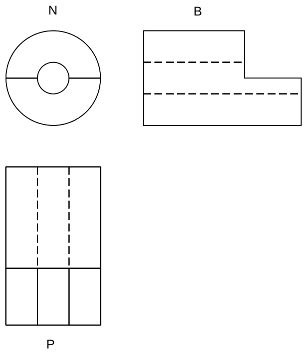
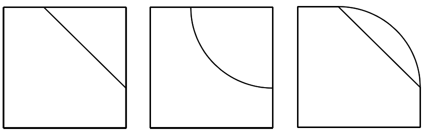
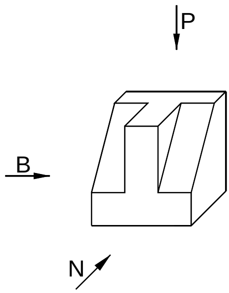


b)

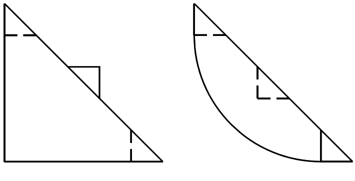
10.

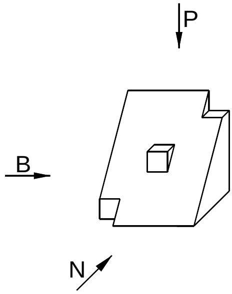
a) *Sample solutions*





b) *Sample solutions*





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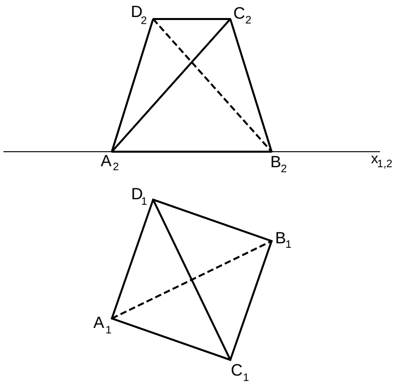
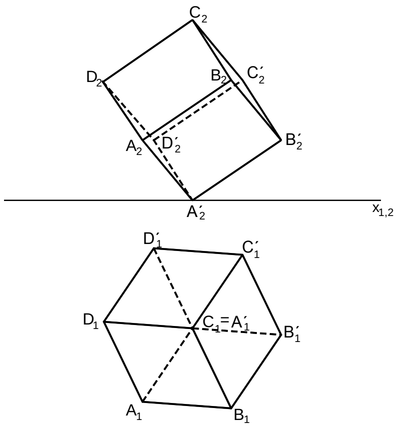


a)



b)

13. AV
14. a), b), c), d), g), h), i), j), k), l), m) and n) are; e) and f) are not
15. 60°
17. 10
18. 10
19. tetrahedron 6, cube 9, octahedron 9, dodecahedron 15, and icosahedron 15.
20. 24
21. 11

Other problems and ideas like Tangram, Origami, the cube Soma etc., can be studied eg. in Steinhaus (1958), Pugačov (1960), Gardner (1968, 1988), Barr (1987), Kuřina (1976), Hejný (1980), Molnár (1986), Opava (1989), Hejný a kol. (1990), Molnár a Kobza (1990 a 1991), Adam a Wyss (1994), Máca a Macků (1996), Šarounová (1998), Leischner (2003), Perný (2004); you can also use the teaching aid of Stopenová (1999), various puzzles and building blocks (eg. Žídek, 1997); and computer games or other programmes can also help.

Unit Title	**My foot and statistics**
Topic	Probability and statistics
Name and email address of person submitting unit	Pavla Žufníčková, olifa@seznam.cz Josef Molnár, josef.molnar@upol.cz
Aims of unit	To show some useful terms of statistics (normal distribution, Gaussian curve, …) and practice using statistics. To work with errors of measurement, to promote cooperation and responsible work in the class, to develop work with text, to develop creativity, and to inform students about the condition of their feet.
Indicative Content	3 lessons To make a footprint (plantogram) on a piece of paper, to find a condition of the arch, to do some other measurements of foot and use it for comparison to Anthropometric tables.
Resources needed	Textbooks used at primary school, papers, pencils, computers.
Teachers notes	

This project has been funded with support from the European Commission in its Lifelong Learning Programme (539234-LLP-1-2013-1-AT-COMENIUS-CAM). This publication reflects the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

My foot and statistics

1. *Lesson:*

Part of motivation:

- Enjoyable test (There are some important terms useful for the next parts).
- Making the footprint on a sheet of paper. You need greasy cream, paper, pen and napkins. Apply cream to the barefoot and stand for five minutes on a sheet of paper. Than take your foot carefully away and trace the footprint. Do the same for your second foot. Write your signature on the papers.

2. *Lesson*

Task for students: Try to find out the condition of your foot. Find the length and the width of your foot as precisely as possible and evaluate them using Ni (normalized index). Your teacher will give you all material. You will attempt the work as though you are an expert in the group.

- Working in a group of experts. (It is a method of learning. Members of expert-groups study short parts of a topic and then they pass on their knowledge to the other members of the group.). The students will determine the work order and any other information needed for working in groups.

You need a text about:

1. The methods of finding the condition of a foot,
2. Working with anthropometrics tables
3. The normal distribution.

- Measuring, counting and working according to result of expert-groups.

You need different types of measuring instruments and a calculator.

3. *Lesson*

Task for students: Do the research in the class on the base of results from the last lesson. Try to use all your knowledge about statistics. Your outcome could be in the form of a diagram or a chart. Try to appraise the area of a footprint.

The things you need and the process depend on the knowledge of the students.

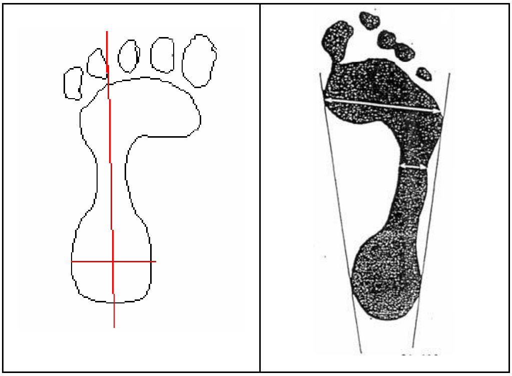
The methods of foot – condition researching

The state of foot vault is described in the plantogram. It gives us information about the type of foot. The types of foot are the flat foot, the normal foot and the high foot. We can use some chemical methods, but they are more complicated than non- chemical methods.

Non-chemical plantograms are made by mechanical printing of somebody's foot onto paper. We can use printing ink or greasy cream. A disadvantage is that the foot is dyed in the process. Therefore anthropologists and specialists use the plantograf. There isn't any contact between the foot and the dye in this method.

Evaluating the plantogram method

Mayer's methods:



Find the widest part of the heel. Find the middle of it. Connect this point with the inner edge of the fourth toe. Draw the straight line as in the picture below. This straight line is called Mayer's line. We use it for the evaluation of the condition of foot arch. If the middle part of the footprint overlaps the Mayer's line, it indicates a flat foot.

Chippauxe's and Šmiřák's methods:

We find the ratio of the widest and the narrowest part of a plantogram. We measure them on the vertical line to the lateral tangent of the plantogram. If there is a gap between upper and lower part of the footprint it indicates a high foot. You must measure the length of this space in this case. There is a picture below:

$$\text{Foot index} = \frac{\text{The smallest measure value}}{\text{The greatest measure value}} * 100$$

Foot condition	Foot index or length of gap
Normal foot	0.1 – 45.0
Minor flat foot	45.1 – 50.0
Medium flat foot	50.1 – 60.0
Bad flat foot	60.1 – 100.0
Minor high foot	0.1 cm – 1.5 cm
Medium high foot	1.6 cm – 3.0 cm
Bad high foot	More than 3.1 cm

Text about Normalized index

Evaluation using NORMALIZED INDEX (N_i)

Formula for counting N_i:

$$N_i = \frac{x_i - x}{\sigma}$$

x_i – Measured number.

$\overline{x}$ – Arithmetic mean of the reference file, which can be found in Antropometrics tables.

σ – Standard deviation, which can be found in Antropometrics tables.

Number N_i shows how much an individual can differ from the arithmetic mean. Zero is arithmetic mean of reference file. Minus sign signals deviation below the arithmetic mean (measured value is lower than arithmetic mean). Plus sign signals deviation above the arithmetic mean (measured value is higher than arithmetic mean).

Evaluation of sign

N_i in interval	Development of sign	Proportion of the population
to ± 1	Average	68
to ± 2	Above - average	95
to ± 3	Below the average	99,7
> ± 3	Pathological disproportion	100

If we measure some characteristics of the human body and if we research the N_i number of these characteristics, we find that most of the values are close to the average. Values, which are very far from the average, are almost impossible. We talk about NORMAL DISTRIBUTION and we can use the Gaussian curve to characterize it.

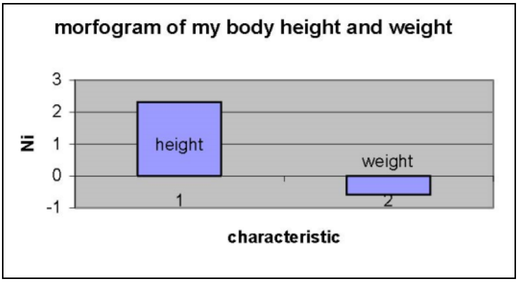
Morfogram of physique

We can make the Morfogram of physique from the Normalized index (N_i). We can specify relative disproportionality of the parts of the human body.

We represent characteristics of the human body on the horizontal axis and an N_i number from –3 to +3 on the vertical axis. Columns of particular length give information about each characteristic.

Example

Histogram



A histogram is the easiest type of column graph to use. You can use rectangles of different size. These rectangles represent relative size of the parts of human body or frequency of characteristics. We represent body characteristics on the horizontal axis and frequency of these or size on the vertical one. If we connect columns using abscissas, we talk about a POLYGON.

Lifelong
Learning
Programme

Unit Title	**Excel in Mathematics Education**
Topic	Calculus
Name and email address of person submitting unit	Ján Beňačka jbenacka@ukf.sk
Aims of unit	Students explore basic skills and knowledge of calculus using applications developed by author. Visualization and dynamical approach is dominant in solving problems. Topics: Graphing functions if D = $\mathbb{R}$ Graphing functions if D $\neq$ $\mathbb{R}$ Maximum and minimum of a function Calculating area, volume and length of curve
Indicative Content	4–8 lessons Age of students: 15–19 years
Resources needed	Excel applications developed by author.
Teachers notes	

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Graphing functions if D = $\mathbb{R}$

The application graphs functions the definition domain of which is $\mathbb{R}$. The graph reacts interactively to changes in parameters, which allows investigating the effect of the parameters on the graph. Once completed, the application can be used as a template for graphing other functions defined on $\mathbb{R}$. The quadratic, sine and exponential function are graphed in Figs. 1-3.

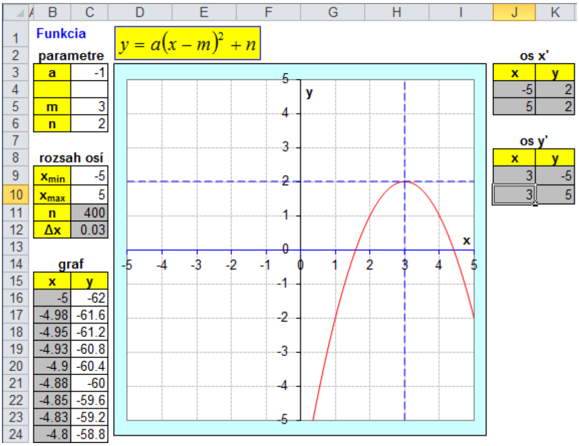

Figure 1: Quadratic function

The white cells contain inputs. The grey cells contain formulas. The parameters of the function are in cells C3:C6. The range of the graph is in cells C9:C10. The chart does not react automatically to change in $x_{\min}$ a $x_{\max}$ therefore the maxima and minima of the axes has to be adjusted manually in the chart, keeping the coordinate system orthonormal.

The graph is of type *xy* line. It is drawn over 401 points (cell C11). The step is in cell C12 calculated by the formula =(C10-C9)/C11. The points of the graph are in range B16:C416. Cell B16 contains the formula =C9. Cell B17 contains the formula =B16+$C$12, which is filled down as far as cell B416. Cell C16 contains the formula =$C$3*(B16-$C$4)^2+$C$5, which is filled down as far as cell C416. The shifted axes *x*’ and *y*’ are of type *xy* line (cells J4:K5 a J9:K10). Cell J4 contains =C9, J5 contains =C10, K4 and K5 contain =C6. Cells J9 and J10 contain =C5, K9 contains =C9 and K10 contains = C10.

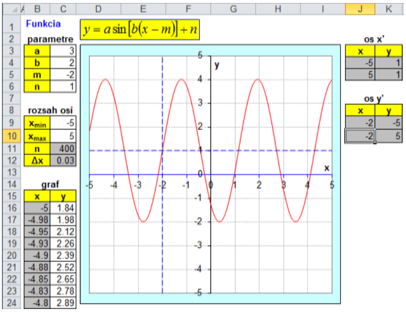

Figure 2: Sine function

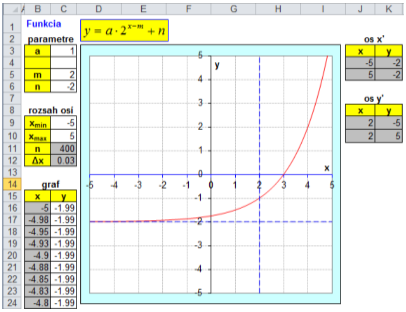

Figure 3: Exponential function

Graphing functions if D ≠ ℝ

The application graphs functions the definition domain of which is not ℝ. The definition domain can be ascertained from the graph. The graph reacts interactively if the parameters are changed, which allows investigating the effect of the parameters on the graph. Once completed, the application can be used as a template for graphing any function. The linear fractional, second root and logarithmic function are graphed in Figs. 1–3. Compound functions are graphed in Figs. 4 and 5.

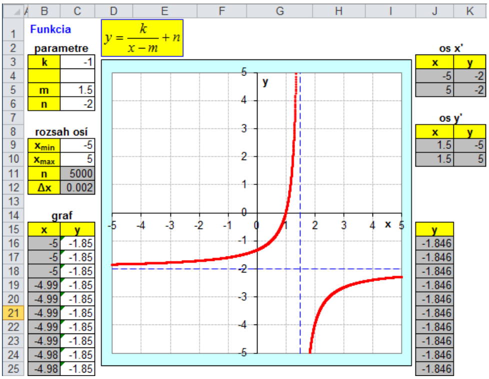

Figure 4: Linear fractional function

The white cells contain inputs. The grey cells contain formulas. The parameters of the function are in cells C3:C6. The range of the graph is in cells C9:C10. The chart does not react automatically to change in x_{min} a x_{max} therefore the maxima and minima of the axes has to be adjusted manually in the chart, keeping the coordinate system orthonormal. The graph is of type *xy* point. It is drawn over 5001 points (cell C11). The step is in cell C12 calculated by the formula =(C10–C9)/C11. The coordinates of the points are calculated in range B16:C5016. Cell B16 contains the formula =C9. Cell B17 contains the formula =B16+$C$12, which is filled down as far as cell B5016. Cell C16 contains the formula =$C$3/(B16-$C$5)+$C$6, which is filled down as far as cell C5016. Cell J16 contains the formula =IF(ISERROR(C16),NA(),C16), which is filled down as far as cell J5016. The graph is drawn over ranges B16:B5016 for *x* and J16:J5016 for *y* therefore the points that are out of the definition domain are not graphed. The shifted axes *x*’ and *y*’ are of type *xy* line (cells J4:K5 a J9:K10). Cell J4 contains =C9, J5 contains =C10, K4 and K5 contain =C6. Cells J9 and J10 contain =C5, K9 contains =C9 and K10 contains = C10.

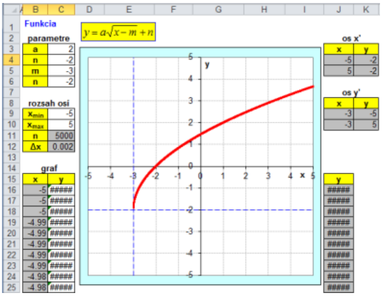

Figure 5: Second root function

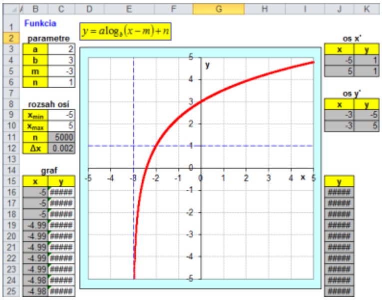

Figure 6: Logarithmic function

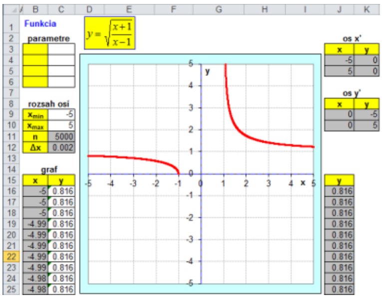

Figure 7: Compound function

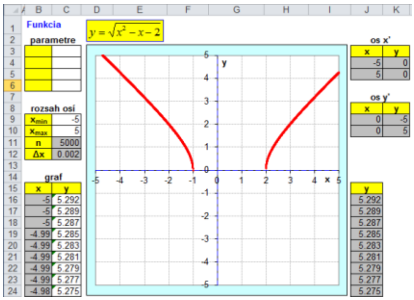

Figure 8: Compound function

Maximum and minimum of a function

The tasks in which extremes of functions are investigated are solved at upper secondary level traditionally within introduction to calculus. With Excel, they can be solved without using derivatives. The following task is resolved in Fig. 9: A rectangular yard is fenced off along a wall, that is, from three sides, by a wire fence of length 100 m. What should be the sides of the yard so that the area is (a) maximum (b) 1 000 m^2?

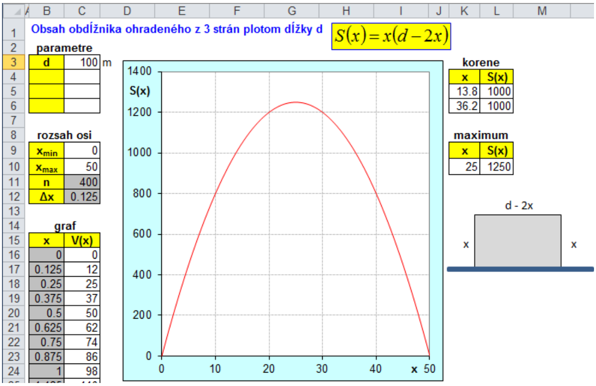

Figure 9: Area of a yard fenced from three sides with a wire fence of length *d*

The white cells contain inputs. The grey cells contain formulas. The parameters of the function are in cells C3:C6. The range of the graph is in cells C9:C10. The chart does not react automatically to change in $x_{\min}$ a $x_{\max}$ therefore the maxima and minima of the axes has to be adjusted manually in the chart. The graph is of type *xy* line. It is drawn over 401 points (cell C11). The step is in cell C12 calculated by the formula =(C10-C9)/C11. The points are in range B16:C416. Cell B16 contains the formula =C9. Cell B17 contains the formula =B16+$C$12, which is filled down as far as cell B416. Cell C16 contains the formula =B16*($C$3-2*B16), which is filled down as far as cell C416.

The task can be solved in two ways. The first one is based on refining the interval around the stationary point or root. I can be seen in Fig. 9 that the stationary point is between 20 and 30, the first root is between 10 and 15 and the second one is between 35 and 40. If 20 and 30 are written in cells C9 and C10, then the step on the *x* axis becomes 0.025. The maximum can be found in range C16:C416. It is $y = 1\,250$ at the accuracy of 0.0012 (Fig. 10). The stationary point is $x = 25$ at the accuracy of the half-step, that is, 0.0125. If values more closely to the stationary point or root are written in cells C9 and C10, then the errors are smaller. The extremes can also be found immediately by Excel functions. Cell L10 contains =MAX(C16:C416), however, there is no information about the accuracy.

The other way of solving the task is with using Excel tools Solver and Goal Seek. Solver is in tab *Data*, Goal Seek is in tab *Data*, *What-If Analysis* (if Solver is not in tab *Data*, then click menu *File*, *Options*, *Add-Ins*, *Solver Add-in*, click button *Go*, tick *Solver Add-in* and click button *OK*). A value close to the stationary point, e.g. 23, is written in cell K10. The function formula is copied from cell C16 and pasted in cell L10. Solver is started. In Solver window, click field *Set Objective* and click cell L10. In section *To*, click *Max*. Click field *By Changing Variable Cells* and click cell K10. Click button *Solve*. To find a root, write a value close to the root, e.g. 12, in cell K10. In Solver window, just change section *To* by clicking field V*alue Of* and writing 1000 in it, then click button Solve. Equations can be solved by the Goal Seek tool, too. It is on tab *Data*, *What-If Analysis*. Write a value close to the root, e.g. 12, in cell K10. In Goal Seek window, click field *Set cell* and click cell L5. Write 1000 in field *To value*. Click field *By changing* and click cell L5. Click button OK.

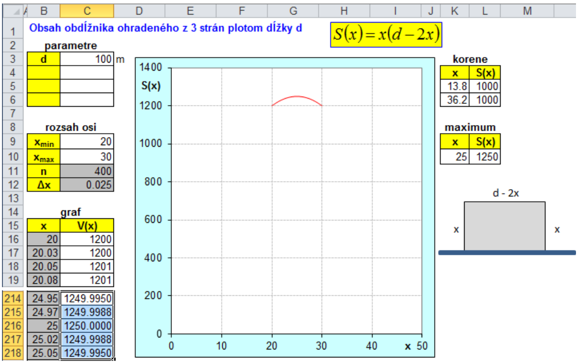

Figure 10: Figure 9 with adjusted $x_{\min}$ and $x_{\max}$

The following task is resolved in Fig. 11: A box is made from a square carton of side length 1m so that squares are cut in the corners and the sides are turned up and stuck together. What should be the sides of the squares so that the volume of the box is (a) maximum (b) 50 litres? The input in cell C3 is in centimetres but the graph is computed in litres. Cell C16 contains the formula =B16*($C$3-2*B16)^2/1000, which is filled down as far as cell C416.

The following task is resolved in Fig. 12: Two corridors of widths *a* and *b* meet at a right-angle corner. What is the maximum length of a rod to get it from one corridor to the other if it is shifted on the floor? Cell C16 contain the following formula, which is filled down as far as cell C416: =$C$3/SIN(RADIANS(B16))+$C$4/COS(RADIANS(B16)).

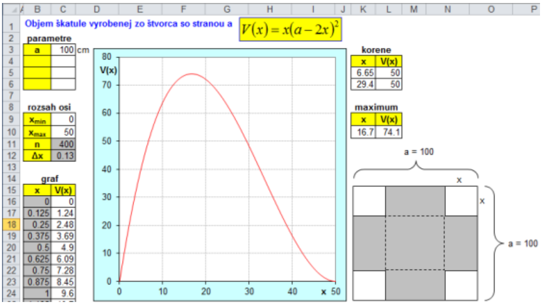

Figure 11: Volume of a box made of a square carton of side *a*

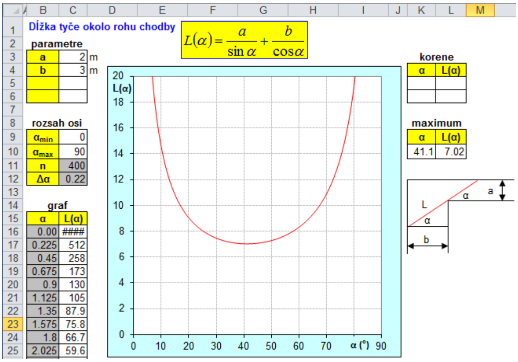

Figure 12: Length of a rod shifted on the floor and touching the walls and the corner of a right-angle corridor

Calculating area, volume and curve length

The applications graph functions and calculate by the rectangular method the area bounded by the graph, volume of a body created by rotating the graph and the length of the graph. The graph reacts interactively to changes in parameters. Once completed, the application can be used as a template. Areas bounded by the quadratic function are calculated in Figs. 13 and 14. Volume of a body created by rotating the graph of the second root function is calculated in Fig. 15. Lengths of the graphs of the quadratic and sine function are calculated in Figs. 16 and 17.

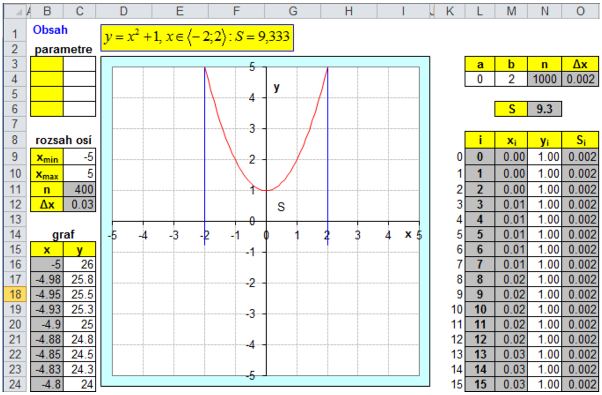

Figure 13: Area

The white cells contain inputs. The grey cells contain formulas. The parameters of the function are in cells C3:C6. The range of the graph is in cells C9:C10. The chart does not react automatically to change in $x_{\min}$ a $x_{\max}$ therefore the maxima and minima of the axes has to be adjusted manually in the chart. The graph is of type *xy* line. It is drawn over 401 points (cell C11). The step is in cell C12 calculated by the formula =(C10-C9)/C11. The points of the graph are in range B16:C416. Cell B16 contains the formula =C9. Cell B17 contains the formula =B16+$C$12, which is filled down as far as cell B416. Cell C16 contains the formula =B16^2+1, which is filled down as far as cell C416.

Cells L4 and M4 contain the bounds of the figure (it is symmetric). The interval is divided to subintervals. The number is in cell N4. The maximum is 1000 (see cell N6). Cell O4 contains the length of the subintervals calculated by the formula =(M4-L4)/N4. Cell L9 contains =0, cell L10 contains =L9+1. Cell M9 contains =L4, cell M10 contains =M9+$O$4. Cells N9 and N10 contain the formulas =M9^2+1 and =M10^2+1. Cells O9 and O10 contain the formulas =N9*$O$4 and =N10*$O$4 that give the area elements. Cell N6 contains =2*SUM(O9:O1009). Cell N4 contains =COUNTA(L9:L1009)-1. Due to the formula, if range L10:O10 is filled down, the number of the subintervals is determined automatically and shown in cell N4, and the correct result appears in cell N6. To diminish the number of the subintervals, it is enough to select the redundant cells and press the key DELETE.

The application in Fig. 15 differs, except for the function, in cells M6 a O8, which contain the symbol for volume, and in cells O9 a O10, which contain the formulas =PI()*N9^2*$O$4 and =PI()*N10^2*$O$4 that give the volume elements. The application in Fig. 16 differs, except for the function, in cells M6 a O8, which contain the symbol for length, in cell O9, which is empty, and in O10, which contains the formula =SQRT($O$4^2+(N10-N9)^2) that gives the length element.

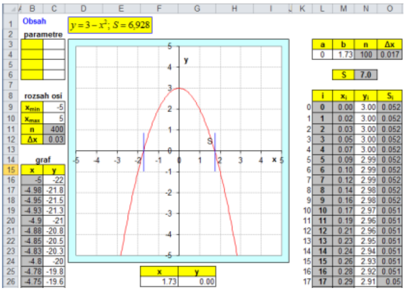

Figure 14: Area

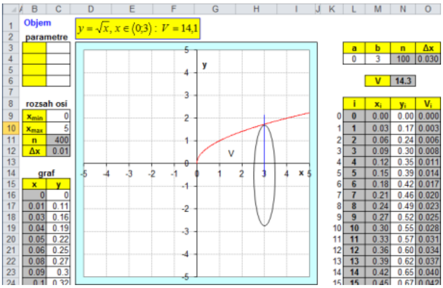

Figure 15: Volume

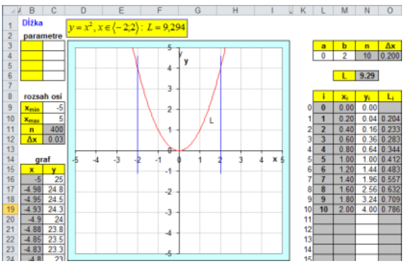

Figure 16: Curve length

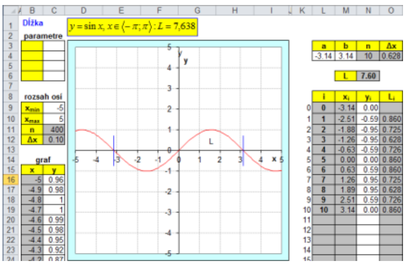

Figure 17: Curve length

Lifelong Learning Programme

Unit Title	**Do you want to be a multi-confectioner?**
Topic	History of Mathematics and Physics
Name and e-mail address of person submitting unit	Pavla Žufníčková, olifa@seznam.cz Josef Molnar, josef.molnar@upol.cz
Aim of unit	To display some parts from the history of mathematics and to develop the creativity of students. Students will be able to make up a test.
Indicative Content	2 lessons To prepare an enjoyable contest and to take part in this contest.
Resources needed	Textbooks and other suitable books, box of confections.
Teachers notes	I would like to add that it is not so important to think up a really complicated question. The enjoyment of the work is more important. The students can create the test from the questions mentioned within the contest, and this might be used in the following lessons.

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Do you want to be a multiconfectioner?

1. Lesson (20 minutes)

Motivation part: A teacher prepares a short lecture about some interesting parts from the history of mathematics. Various materials could be used depending on the teacher. At the end of this lecture the teacher invites the students to enter a competition dealing with the history of mathematics.

Task for homework: Look up some material for the competition. You can use books, web sites or any other data sources. Think up some problems dealing with the history of mathematics.

2. Lesson

Competition preparation:

1. Class is divided into two groups. Each group works separately at this stage. (If there are too many students in the class, they can be divided into three groups and the work will be similar).
2. Each student thinks about a question, using his or her homework results, and writes down one question and four answers. Only one of the answers is right.
3. The members of each group read their questions together. They range from the easiest to the most complicated type of question.
4. Each group chooses one or more speakers.

Competition:

1. The teacher starts the competition (it is possible to use any popular competition on TV as a model).
2. The speaker of the first group asks the questions; first the easiest one and the most complicated at the end. The second group considers these and produce answers together. The speaker from the second group answers. If they pass, they receive one confection (or more). If they fail, they get nothing. They can use help (for example: if they don't know the answer, the author of the question sets aside two wrong answers. This can be used only once in the game)
3. The groups change their roles when all questions are asked.

The teacher finishes the competition.

SCIENCE

Unit Title	**Viscous flow**
Topic	Mechanics
Name and e-mail address of person submitting unit	Renata Holubova renata.holubova@upol.cz
Aims of unit	The unit describes the basic laws in fluid dynamics. Interdisciplinary relationships are shown - e.g. blood transport. A laboratory experiment is added.
Indicative Content	Viscous flow, turbulent flow, Poiseuille's law, Reynolds number.
Resources needed	Access to web.
Teachers notes	The unit includes a laboratory experiment. The practical activity leads to the understanding of the law and its use in technology and medicine. Interdisciplinary relationships are pointed out. Further applications can be shown – e. g. transport of pollutants.

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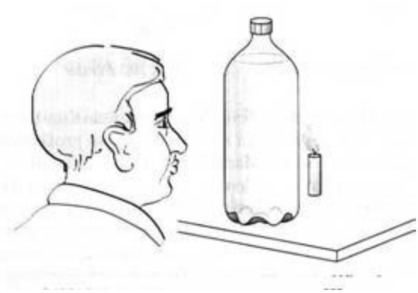
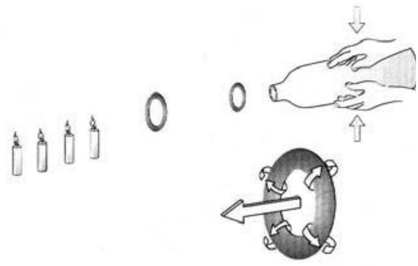
Viscous and turbulent flow

Motivation

Tornado in a bottle: How can I get the water from the upper bottle to the lower one?

Air whirls: Put out the candle!





Main Activity

Viscous Flow

The idea of an ***ideal flow*** – every layer moves with the same velocity, there is no viscosity. The velocity in the middle of the pipe is the same as in layer next to the wall.

In a ***real fluid***, where viscosity is present, the velocities are not the same – in the middle of the pipe is the greatest velocity, the fluid layer next the wall has a velocity near zero.

How we will express, what viscosity is?

Let´s have two parallel plates. The top plate is free to move, the bottom one is stationary. If the top plate is to move with a velocity $\boldsymbol{v}$ (relative to the bottom plate), a force $\boldsymbol{F}$ is required. Another force is needed in the case of water or in the case of honey or glycerin. A fluid can be modeled as a lot of plates with various velocities. The velocity of each layer is different. The greatest one is on the top, at the bottom it is zero. This kind of flow is a *laminar flow*.

The tangential force ***F*** required to move a fluid layer at a constant speed *v*, when the layer has an area *S* and is located a perpendicular distance y from an immobile surface, is *given by*

$$F = \frac{\eta S v}{y}$$

where η is the coefficient of viscosity. SI unit for viscosity: Pa · s

Common unit: *poise* (P) **1 poise (P) = 0,1 Pa · s**

Jean Poiseuille (1797–1869) – French physician, he explored the movement of fluids in pipes to acquire laws or regularities of the blood flow in our body.

Values of viscosity

Under ordinary conditions the viscosity of gases is smaller than those of liquids. The viscosity depends on temperture – the viscosities of liquids decrease with higher temperature, the viscosity of gases increase as the temperature is raised.

Viscosity:

Water (20 °C)	$1{,}00 \cdot 10^{-3}$ Pa · s
Benzene C_6H_6	$0{,}65 \cdot 10^{-3}$ Pa · s
Ethanol C_2H_6O	$1{,}20 \cdot 10^{-3}$ Pa · s
Glycerol $C_3H_8O_3$	$1480{,}00 \cdot 10^{-3}$ Pa · s
Blood (37 °C)	$5{,}00 \cdot 10^{-3}$ Pa · s
Air (18 °C)	$0{,}019 \cdot 10^{-3}$ Pa · s

Viscous flow is common in various situations – for example oil moving through a pipeline.

We have to identify factors that determine *the amount of liquid,* that flows across a cross-section of a pipe in a time interval. This is the ***volume flow rate*** *Q* (in m^3/s).

Q is proportional to $P_2 - P_1$ – the pressure difference between any two locations along the pipe (greater pressure leads to a larger flow), a long pipe offers a larger resistance to the flow than a short pipe does (pumping station along the long pipelines). Height viscosity fluids flow less readily than low viscosity fluids. The greatest importance has the dependence on the radius r of the pipe – Q is proportional to the fourth power of the radius (r^4). The mathematical relation is known as ***Poiseuille´s law:***

A fluid whose viscosity is η, flowing through a pipe of radius r and length L , has a volume flow rate Q given by

$$Q = \frac{\pi r^4 (P_2 - P_1)}{8\eta L}.$$

P_2 and P_1 are pressures at the ends of the pipe.

The central formula in the laminar flow model is the **Poiseuille equation***:*

$$R = \frac{8\eta L}{\pi r^4}$$

Further simplification: the Ohm law

For clinical applications fluid properties may be taken as constant. The Poiseuille equation can be simplified to resemble the Ohm law of electrical resistance. This law relates voltage drop across an electric circuit (ΔU) to electric resistance (R) and electric current (I), as follows:

$$\Delta U = IR.$$

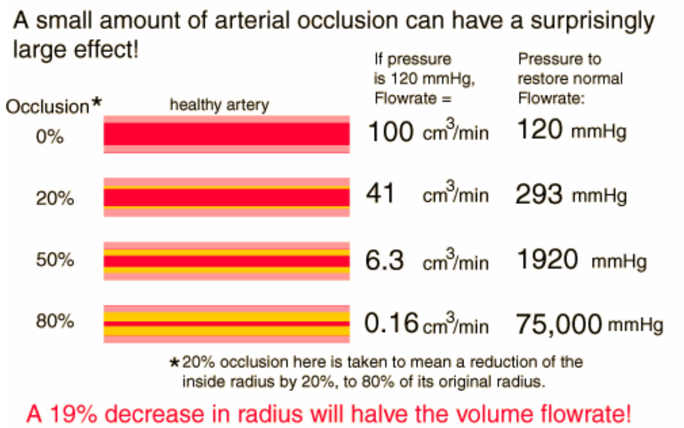
The greater the current (electron flow) or resistance, the higher the voltage drop required. For the sake of simplicity, ΔU often is written as U, although it is the change in voltage and not the absolute voltage that matters.

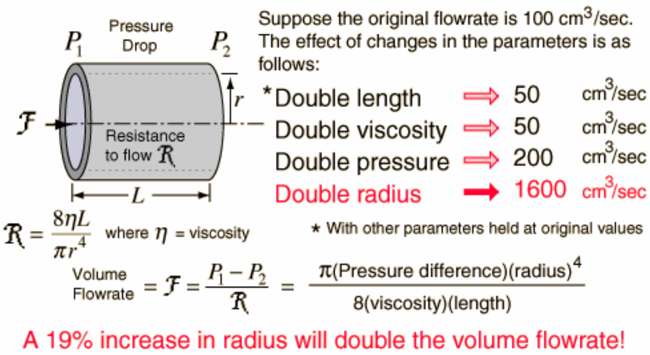
Fluid-flow calculations can be modelled as simple electric circuits. For this analogy (model) we relate ΔU to ΔP, defining flow resistance as $R = 8\,L\eta\,/\,\pi r^4$ to obtain the following:

$$\Delta P = QR$$

Resistive losses (pressure drops) linearly are related to flow rate and flow resistance. While flow resistance linearly relates to conduit length, it is inversely related to the fourth power of the radius (or diameter). For example, a 1cm–diameter pipe has 16 times the flow resistence of a 2 cm–diameter pipe of the same length and carrying the same fluid.

Interdisciplinary relations





Turbulent flow

Turbulent flow is when particles in a fluid move in many *different* directions and at many different velocities. Turbulent flow is the opposite of laminar flow.

Next question – when it comes to overlapping from laminar to turbulent flow – ***Reynolds´ number***. The Reynolds´ number is a dimensionless constant *R*, where

$$R = \frac{\rho r v}{\eta},$$

where v is the velocity (critical velocity). For cylindrical pipes, the Reynolds number corresponding to the critical velocity is about 2 000.

Thus for water flowing through a pipe of diameter 2 cm (for example garden hose), the critical speed is

$$v_\mathrm{c} = 2\,000 \frac{1 \cdot 10^{-3}\,\mathrm{N \cdot s/m^2}}{10^3\mathrm{kg/m^3} \cdot 0{,}02\mathrm{m}} = 0{,}1\,\mathrm{m/s} = 10\ \mathrm{cm/s}.$$

(A low speed, usually the flow is turbulent, it has v = 1 m/s.)

Problems

1. A pressure difference of $1{,}5 \cdot 10^3$ Pa is needed to drive water through a pipe whose radius is $6{,}8 \cdot 10^3$ m. The volume flow rate of the water is $3{,}2 \cdot 10^{-3}$ $\mathrm{m^3/s}$. What is the length of the pipe? The water viscosity $\eta = 1 \cdot 10^{-3}$ Pa · s.

2. A blood vessel is 0,1 m in length and has a radius of $1{,}5 \cdot 10^{-3}$ m. Blood ($\eta = 4 \cdot 10^{-3}$ Pa · s) flows at a rate of $1 \cdot 10^{-7}$ $\mathrm{m^3/s}$. Determine the difference in pressure that must be maintained between the two ends of the vessel.

3. Calculate the highest average speed that blood could have and still part in laminar flow when it flows through the aorta ($R = 8 \cdot 10^{-3}$ m, $\rho = 1\,060\ \mathrm{kg \cdot m^{-3}}$).

Laboratory experiment

The purpose is to study viscous fluid flow. The resistance to flow of single capillaries, 2 capillaries in a series and 2 capillaries in a parallel configurations is measured and compared to predictions.

Equipment: 2 cups, 1 panel with 3 glass capillaries (various diameters), 2 identical capillaries in series and in parallel, tubes with valves, 1 beaker, 1 ruler

Introduction: We study the flow of viscous fluids through glass capillaries. The flow rate through the capillary is proportional to the pressure difference across the capillary $\Delta P = RQ$, where R is the resistance of the capillary to the fluid flow

$$R = \frac{8\eta L}{\pi r^4}$$

A beaker is positioned a height h above the cup. The capillary is kept horizontal, and connected with the overflow in beaker. Then the pressure difference on the capillary is $P = h\rho g$, where ρ is the density of water. The water flows in a calibrated beaker, or the mass of the water is determined on laboratory weights. By measuring the collection time, the flow rate Q can be computed. The water level in beaker has to be constant. The length of the capillaries is to be determined.

Various measurements can be obtained:

- measurement with capillaries of the same diameter but various length,
- measurement with capillaries of equal lengths and various diameters,
- one capillary in different height (different pressure).

The other conditions will be constant (temperature, density of water, humidity etc.).

In the second part of the lab experiment is to measure 2 equal capillaries connected in series and parallel (analogy with Ohm law).

Futher reading:

Animations on www:

http://www.physik.uni-wuerzburg.de/physikonline.html

https://www.youtube.com/watch?v=KqqtOb30jWs

https://www.youtube.com/watch?v=eIHVh3cIujU

http://pokusy.upol.cz/iga/iga-2013/fyzika-netradicne/vibracni-viskozimetr-10/

Unit Title	**Energy in Food**
Topic	Energy
Name and email address of person submitting unit	Soňa Čeretková, Soňa Švecová, Janka Melušová sceretkova@ukf.sk
Aims of unit	To explore why and how food is important for the flow of energy in the human body and find out how much energy there is in food.
Indicative Content	2 lessons age od students: 14-16
Resources needed	Internet. Computer.
Teachers notes	Developing the student inquiry and creativity. Students discover how much energy there is in food and how much energy a they need to consume per day. The applet deals about the energy in certain samples of food. http://www.compass-project.eu/applets/1/index_EN.html Students work in small groups, analyse and compare their findings. Groups are asked to prepare posters or presentations about their findings.

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Energy in Food

Note: The unit is reworked part of more complex material available on http://compass-project.eu/resources_detail.php?UG_hodnota_id=5

Introduction

There is much concern that young people today are becoming obese at an early age because of what they eat.

Many countries across Europe are very concerned about the situation and are controlling what school pupils eat in the meals they provide in schools.

Food and cooking is discussed in media daily. Internet provides a lot of information about different types of diet.

First lesson

The first lesson opens the problem of energy in food. Students discuss the topic together, teacher moderates the discussion.

The problem is that we all need a daily intake of energy from what we eat to provide the energy we need for what we do. If we do a lot of exercise on a particular day then we will need more energy than on a day that we spend relaxing.

Pupils discuss possible answers to the question: *How do you find out how much energy is stored in different types of food?*

In this task the method for measuring how much chemical energy is stored in different types of food will be shown to students.

Mathematical content is proportionality. Scientific content of the unit is: energy, calorimeter, calorie, joule.

The goal of the first experiment is to determine the amount of chemical energy stored in food by burning it and capturing the heat given off by simulating an experiment.

Activity 1 for students: Brainstorming

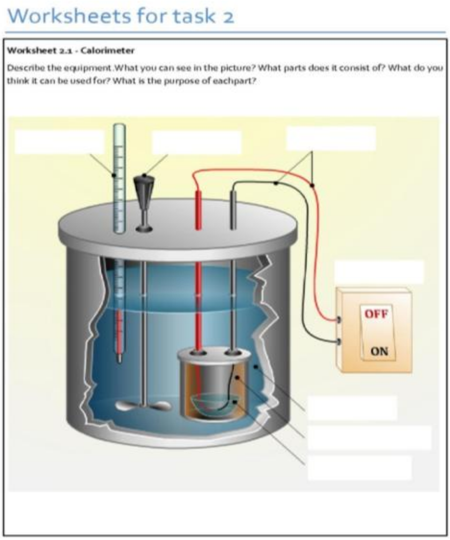
Based on the information “One calorie will raise the temperature of 1g water by 1°C” try to find a way how to measure the energy captured in food.

The basic idea of a calorimeter is to capture the released heat energy with a reservoir of water, which has a high capacity for absorbing heat. The temperature of the water reservoir is measured at the beginning and at the end of the experiment. The increase in the temperature (in °C) times the mass of the water (in g) will give you the amount of energy captured by the calorimeter, in calories.

Activity 2 for students

www.compass-project.eu/applets/#1

In groups of 3-4 describe the equipment you can see in the picture. What parts does it consist of? Can you construct similar one?



Activity 3 for students

Observe how the amount of released energy changes according to the amount of food burned. Do all kinds of food release the same amount of energy? Why not? Which contains the most energy and which the least? (see worksheet 2.2)

Homework: Experiment with applet. Find some other sources of information about energy in food you intake (labels on products, internet,...) Prepare your „food energy diary“ per one day or per several days as a poster or presentation.

Worksheet 2.2 - Experiment

Observe how the amount of released energy changes depending on the amount of food burned. Do all food types release the same amount of energy? Why not? Which contain the most energy and which the least? Given that 1 calorie equals 4.184 J, calculate the energy in Joules as well.

Food	*Amount (g)*	*Initial temperature (°C)*	*Final temperature (°C)*	*Difference (°C)*	*Energy (cal)*	*Energy (kJ)*
Bread	0.2					
Bread	0.4					
Bread	0.6					
Bread	0.8					
Bread	1.0					
Cereal	0.2					
Cereal	0.4					
Cereal	0.6					
Cereal	0.8					
Cereal	1.0					
Peanuts	0.2					
Peanuts	0.4					
Peanuts	0.6					
Peanuts	0.8					
Peanuts	1.0					

The second lesson

Students work in small groups (3-4 students in one group). They discuss and compare their individual findings prepared at home. Students analyse their individual „food energy diaries“, their individual energy intake. The outcome of the lesson is a group presentation of the results about food and energy. The whole class discussion could lead to healthy food consumption. Students can prepare the proposal of their own lunch menu for the school cantine, ask the cantine staff for cooperation and cooking the menu. Students can prepare a survey about the popularity of their menu among all students of the school, who eat menu in the school cantine.

Lifelong Learning Programme

Unit title	**Radioactive Nathanium**
Topic	Atomic and Nuclear
Name and email address of person submitting unit	Graham Tomlinson olliecat@onetel.net
Aims of Unit	The unit gives data for a fictitious radioactive element Nathanium, and this requires students to do an elementary analysis to find its half-life, and guides the student to graphically analyse the data using log graph methods.
Indicative content	Count rates, half-life, log graphs.
Resources needed	Pupils will need a copy of the worksheet either in hard copy or electronic form; they will also need a calculator and graph paper.
Teachers notes	This activity is a suitable main activity and would require about 35 minutes to complete. It is suitable for 14+ pupils. It would also be suitable for a homework activity. Learning outcomes for this activity All pupils will be able to calculate the rate per minute given the data provided. Most pupils must be able to Calculate the log of this data and plot this data onto a suitable graph. Some pupils will be able to use the formula provided to calculate half-life.

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Radioactive Nathanium

The newly discovered **super super-heavy** isotope nathanium–659 is found to be radioactive and a small sample has its count rate measured at intervals of 1 minute. The background count in the environment where the Nathanium is measured has already found to be 32 per minute.

Time in minutes t	Measured count rate per minute	Corrected count rate per minute, A	$\ln A$
0	787		
1	633		
2	525		
3	401		
4	316		
5	262		
6	223		
7	175		
8	152		
9	121		
10	105		
11	88		
12	76		

To analyse the results the obvious thing to do first is to plot a graph of corrected count rate and time.

Plot this graph with corrected count rate A on the vertical axis and time t on the horizontal axis.

From the graph, estimate the half-life of Nathanium.

A better way to calculate the half-life is to use the following method. We start with a relationship between ln A and t, which you should be able to find derived in any standard text book at this level.

$$\ln\left(\frac{A}{A_0}\right) = -\lambda t$$

In this equation A_0 is an initial count rate which we can ignore for this analysis, and λ is the radioactive decay constant for Nathanium.

This can be written,

$$\ln A - \ln A_0 = -\lambda t$$

Now proceed as follows, rearrange the equation with ln A as the subject and compare it with the standard equation for a linear relationship $y = mx + c$ plot a new graph with ln A on the y-axis and t on the x-axis – what will the gradient of this graph represent?

Find a value for λ from your graph – this is a measure of the radioactivity of Nathanium. To find its half-life use the relationship,

$$t_{1,2} = \frac{\ln 2}{\lambda}.$$

NOTE: Nathanium-659 does not exist, but if it did this analysis would work and is applicable to all known radioisotopes.

Lifelong
Learning
Programme

Unit Title	**Move it! Dynamic Geometry Software in optics**
Topic	Optics
Name and email address of person submitting unit	Andreas Ulovec Andreas.Ulovec@univie.ac.at
Aims of unit	Using dynamic geometry software for showing the refraction of light in a drop of water and in an optical lens
Indicative Content	Optics, refraction
Resources needed	Computer with GeoGebra
Teachers notes	

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Move it! – dynamic geometry software in optics

In optics when it comes down to show the path of rays of light through glass, lenses or systems of lenses, many physics teachers groan – the experiments are quite complex, and you need a lot of equipment. It is difficult enough to show a ray of light in air – you need smoke, dust or any other way of making light visible. To show the path of light in materials, you need special equipment – smoke glass lenses etc. Now that's not always available, and adjustments to the system can usually only be done be removing one piece and putting another piece in. To see what happens if you make a lens thicker, you have to take out the current lens and put in the new one. Students can then observe the situation before the change and after the change – but it is not exactly a gradual change that lets them observe how the path of light actually changes. We want to demonstrate how you can show the path of light through a lens with the help of dynamic geometry software (DGS).

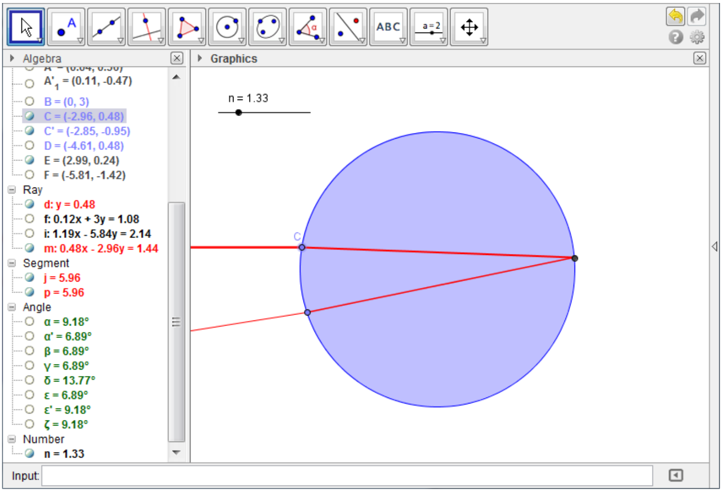
DGS allows us to make constructions with geometric objects like points, lines, etc. Contrary to normal drawing software, the objects maintain their relationships with each other if the position of one of the objects is changed, e.g. you can construct Euler's line of a triangle, then change the position of one of the corners, and the constructed line will remain to be Euler's line of the new triangle. We will use this dynamic property to construct the path of light through a lens whose thickness, radius and index of refraction you can change, and observe the corresponding changes of the light path.

There are several DGS systems available – we chose to use GeoGebra (available at http://www.geogebra.org) due to its easy and icon-oriented user interface and its good term handling abilities.

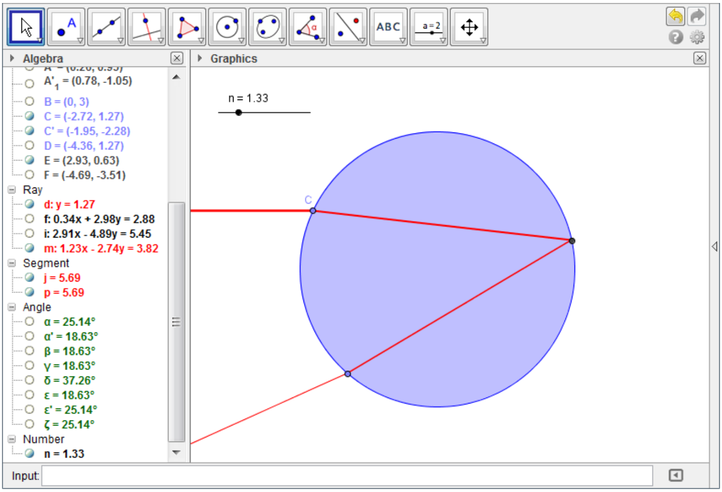
The programs described herein are available in a ready form. They can be used "as is" or – with interested students – also reprogrammed or extended.

Program 1: Refraction and reflection in a drop of water.

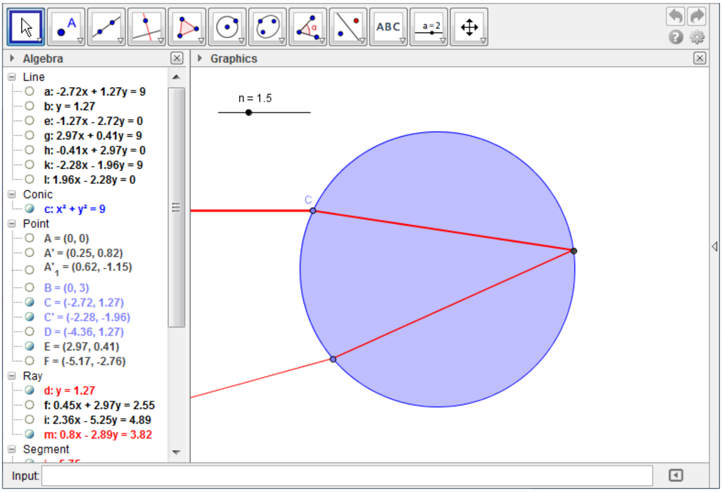
This shows the third-order ray of light (the ray resulting from one refraction, one reflection, and another refraction) in a drop of water. Its refractive index is 1.33:



By pulling on point C you can change the position of the incoming ray:

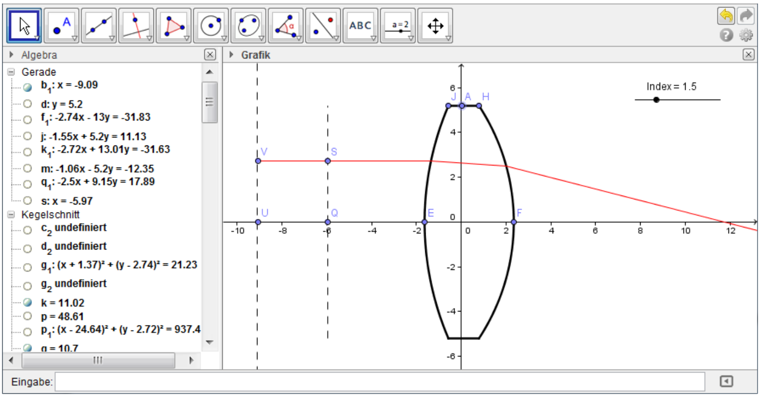


By means of the slider you can also change the refractive index, e.g. to check out another substance:

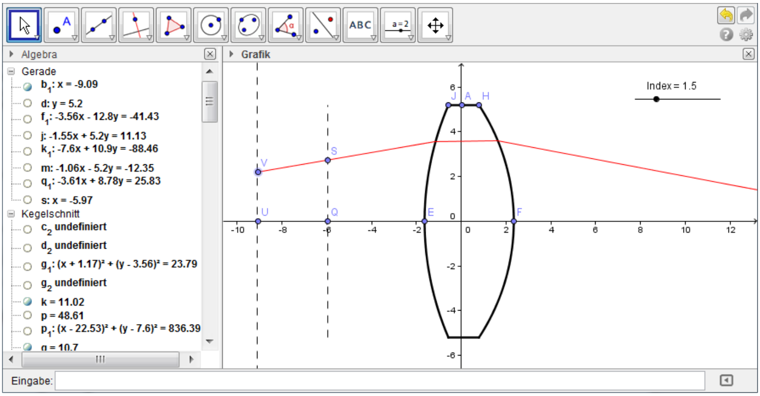


Task: What would happen, when the refractive index is n = 1? Check your ideas with GeoGebra.

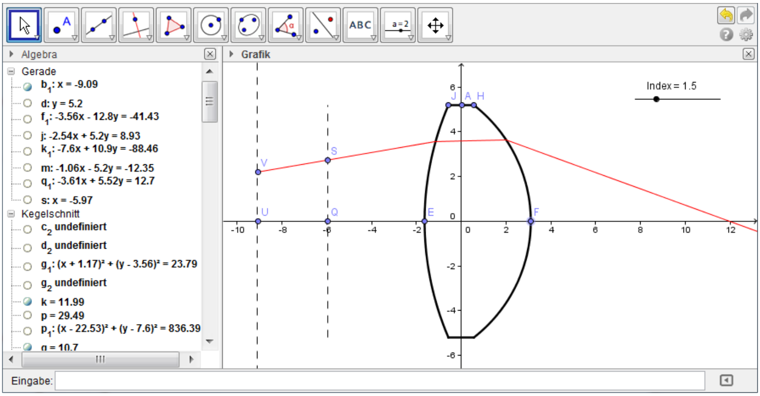
Program 2: This program shows the path of light within a lens:

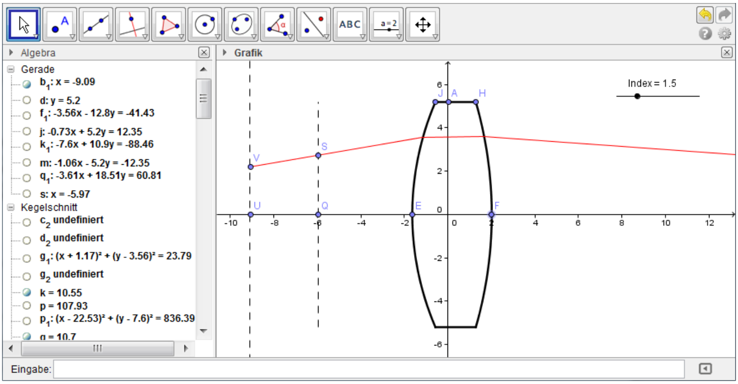


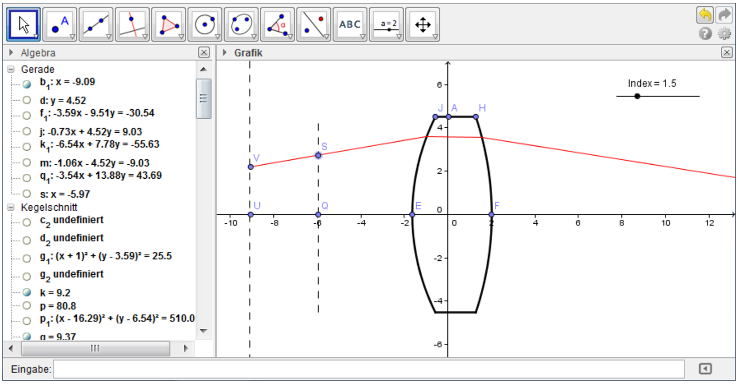
The student can vary the incoming ray by pulling on the points V and S:



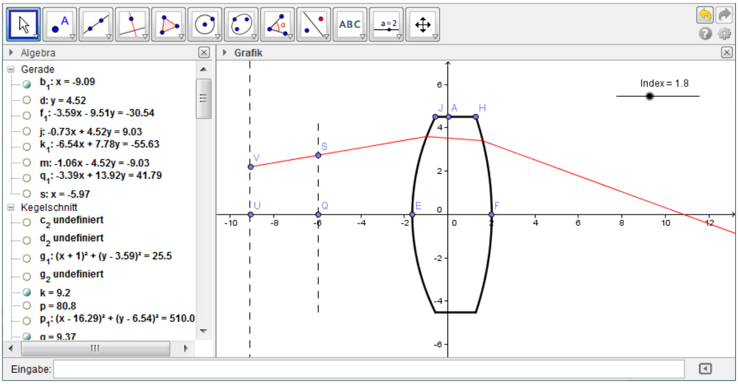
The radius of the two surfaces can be changed by either pulling the points E and F (to make the base thicker or thinner) or the points J and H (to make the border thicker or thinner). The diameter of the lens can be changed by pulling the point A:







Finally the index of refraction may be changed by using the slider:



This program allows the students to experiment with various forms of lenses and values of refractive indices without having to handle sensitive glass pieces or manipulating a lot of objects. The experimental phase can be free or guided. Some possible questions:

Question: What happens if you change the thickness of the lens so that you get a normal (plane) glass?

Question: What happens if you change the index of refraction to 1?

Question: What happens if you send a ray of light along the axes?

Task: Measure the focal length for various forms of the lens.

Lifelong Learning Programme

Unit Title	**Stand by to waste energy**
Topic	Energy
Name and email address of person submitting unit	Gudrun Dirmhirn gudrun_dirmhirn@gmx.at
Aims of unit	Saving energy in the household by knowing the energy consumption of electric household appliances in standby mode
Indicative Content	Natural resources
Resources needed	None.
Teachers notes	None.

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Stand by to waste energy – work sheet (students)

Energy consumption of households generally increases – in Austria as well as in any other highly industrialized country. The number of electric household appliances, the number of electronic and digital equipment, and also the number of households (particularly one-person-households) increase – all adding to more and more energy being consumed.

Question: As we know, energy can not be "consumed" or "wasted", but it remains constant in a closed system. Why do we talk about "wasting energy" or "energy consumption" nevertheless?

We all have a lot of electric and electronic equipment at home. Some of it we do not use too often – the blender, the vacuum-cleaner, the bread-baking-machine. When they are not in use, they are usually switched off, disconnected from electric power, and stored away. But some of the equipment we use almost daily – the TV, the DVD- or video-player, the computer, the satellite-receiver, the radio, the CD-player, the printer, the answering-machine etc. So most of these are always plugged in, always waiting for us to work with them – and normally not completely switched off, but in stand-by-mode. This is usually shown by a little (usually red) light that's on. Now, that tiny red light could not use much energy – or could it?

Task: Find out how many pieces of equipment are in stand-by-mode in your home. How many hours every day are they in stand-by?

Probably you found quite a lot of machines with the little red light in your households. Now, the red light alone would probably not use up much energy. But of course there must be more "on" then just the red light – how else could your TV react when you press the button on the remote? So, every piece of equipment that's in stand-by-mode uses energy – for the remote control receiver, for showing the time, for remembering your last DVD, for storing the last phone number. And if you think about it – how often do you really need one of these features?

The energy consumption in stand-by-mode differs from machine to machine. Newer devices are normally more efficient than older ones.

Task: Look at your stand-by-devices at home and try to find out how much energy they use in stand-by-mode. Look in the manual, the internet, or ask sales people in stores.

Of course you could save energy by reducing your daily TV consumption by one hour, watching one DVD less per week, or not listening to the latest hits on CD one more time. Yes, we know, you have heard that about a dozen times from your parents. And, yes, that would be a very good method to save energy. But for starters you could try this: Disconnect every piece of equipment whose stand-by mode you do not really need from the power supply. Some devices have buttons to switch them completely off (most TV's have), while with others you probably have to pull the plug. You can still watch TV or a DVD whenever you want – just walk to the player and push the button back in or plug it in again. You can save energy – without much changing of behaviour or loss of convenience. You think that's not worth it and adds up to nothing? Let's see!

Task: Find out how much money you can save in a year by switching devices completely off (or disconnect them from power) instead of using the stand-by mode. The figure shows the average energy consumption for some devices. Find out the consumption in kWh and call your power company (or look the information up in the internet) for a price quote

Average energy consumption of devices in stand-by-mode		
Device	Energy consumption (W)	Stand-by-time per day (hrs)
TV (new)	1 – 4	20
TV (old)	10	20
Receiver	5	23
Video-player	4	23
DVD-player	1 – 3	20
Radio	3 – 7	19
Computer	5	20
Screen	2 – 5	20
Printer	3 – 6	23
Cordless phone	1	23
Phone adapter	< 1	23
Answering machine	< 1	24

A study showed that an average Austrian household can save about 37 € (50 US$) every year by just switching off stand-by-devices! If that was to be done by every Austrian household, the energy saved would add up to an amount of 900 million kWh. That's the electricity annually produced by an entire power plant!

Stand by to waste energy – info sheet (teacher)

This project can be done without much effort and shows some surprising results (one of them is that they can use maths and physics to save money). It gives room for discussions about the difference between energy, work, electricity, voltage, and other terms that are likely to be confused in everyday life. It also gives them the opportunity to think about other ways of reducing their energy consumption or general environmental protection. The tasks can also be tackled in group work and do not need to be restricted to their own households. Possible extensions would be:

Task: Look at stand-by-devices at school. Find out how much money the school could save by switching them off completely.

Task: Make a list of stand-by-devices whose stand-by-function you use a) daily b) a couple of times per week c) a couple of times per month d) rarely or never.

Task: Make a survey amongst friends or family of how much energy (or money) they guess can be saved by switching off stand-by-devices.

Task: Take pictures of stand-by-devices and find out how much energy they use in stand-by-mode and how much money can be saved by switching them off completely. Design posters with this information and put them on display in the school building.

Lifelong
Learning
Programme

Unit title	**Build a Space Dictionary**
Topic	Astronomy
Name and email address of person submitting unit	Michele Francis Michelefrancis@washington15.freeserve.co.uk
Aims of Unit	This is a simple exercise in extending student's vocabulary about key words relating to space. Three words are given with their meanings, and then the student has to give words for stated meanings and meanings for stated words. The exercise concludes by asking students to extend the dictionary with words and meanings they give.
Indicative content	Improving vocabulary and definitions related to general space terms.
Resources needed	Text books related to space and astronomy.
Teachers notes	This activity is ideal for improving pupils' vocabulary and assessing their understanding of key term in Astronomy. It is suitable for pupils aged 11 + and although open ended would be good activity for a whole lesson of 1 hour or perhaps for a homework. Learning outcomes for this activity All pupils should be able to define the key words given or identify the key words from the definition given. Most pupils should be able to identify a range of important terms and find definitions for them. Some pupils will be able to discover the deliberate errors and offer corrections to them.

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Build a Space Dictionary

Your task is to build a space dictionary by choosing astronomical keywords linked to a given meaning, then writing what given keywords mean. You must be as precise as possible and use books or internet sources to help you with the task. Three completed examples are given as a guide. You should then complete this part of the task by giving a further six astronomical keywords and their meanings.

Astronomical Keywords	Meaning
astronomy	scientific study of the Universe
star	a ball of gas shining by its own made energy
constellation	a pattern of bright, visible stars
	an instrument used by astronomers
	the path of a planet around the Sun
	the Earth's natural satellite
	a cloud of gas between stars
	shapes of the Moon seen from the Earth
	all stars evolve through this state
	the material between Mars and Jupiter
planet	
atmosphere	
cluster	
white dwarf	
galaxy	

Now give six keywords of your own with their meaning.

This is your final task in building your space dictionary. The sentences below all contain a mistake. Underline the mistake and put the correct statement in the box alongside.

Neptune is a gas giant and is about the same size as Pluto	
Uranus, like Mercury, has a ring system	
Betelgeuse is a young, red star	
The asteroids lie mainly between Earth and Mars	
Mercury has the highest surface temperature of all the planets	
The Solar System is nearly as old as the Universe	
Pluto is difficult to observe as it is so close to the Sun	
The Solar System is close to the centre of the Milky Way galaxy	

Lifelong Learning Programme

Unit Title	**Boltzman law**
Topic	Thermodynamics, energy
Name and email address of person submitting unit	Daniela Horváthová dhorvathova@ukf.sk Mária Rakovská mrakovska@ukf.sk
Aim of unit	Practical test of theoretical law
Indicative Content	2 lessons age of students 17
Resources needed	prepared by author ICT: PC, Excel
Teachers notes	Practical verification of Boltzman law about the division of energy. Application of basic knowledge from thermodynamics in an experiment. Supporting text available for practical physics.

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To basic knowledge about the graphs of functions in teaching physics

Introduction

One of the basic requirements of current educational systems in sciences is to develop in students such abilities which would prove to have a permanent value and a universal application. Such personal abilities doubtless include the understanding of causal relations and their mathematical description by means of functions and their graphs. The graph of a function provides a lot of information which, moreover, can be mediated by a computer.

In the computer era, the presentation of functional relations by means of a graph has become a common means of communication not only in physics and technology, but in our daily life as well. The methods which allow us to express various causal relations and consecutive changes, are permanent values which can be used by young people in different professions. In the paper there is presented a basic knowledge about the graphs of functions which are needed for the activities of future teachers in a physical laboratory by means of a computer-aided block scheme.

1 On the methodology of the formation of abilities in the application of the graph of a function with a physical content

For students, the transfer of knowledge of the graph of a function from mathematics to physics is demanding, which has also been confirmed by research [1], [2]. The graph of a physical function, unlike the mathematical function it specifically features, depicts natural laws and students should know how to find them out. For this reason a special methodology has been developed for the students, future teachers, so that they can master the work with the graph of a physical function. This methodology might come in useful because it can help the pupils of primary or secondary schools to obtain the knowledge in the field

The methodology of the transfer of knowledge about graphs of functions includes:

a) the determination of the extent of the required information by arranging it into a hierarchical line

b) an explanatory teaching text,

c) a block scheme for examining physical dependency through a graphical method using a computer with included information for the student activities.

Students have become familiar with these activities at the beginning of their study through lectures, seminars and laboratory measurements.

Hierarchically arranged structural elements of the graphical method used in the teaching of physics and adjusted to suit the needs of students, future teachers:

- ❑ to approximate the points displaying the results of measurement of a physical action by means of a continuous line of the graph,
- ❑ to use graphic interpolation and extrapolation to determine values of quantities of which the measurement did not take place,
- ❑ to see, on the linear graph actions taking place uniformly, to see connection between the linear graph of the uniformly physical action and the graph of a linear function in mathematics, to know from the graph how to write down the physical equation,
- ❑ to determine the rate of change of physical quantity by the measurement of quotient $\Delta y / \Delta x$ for the graph of function $y = y(x)$ and to see the connection with the slope of the straight line of a linear function in mathematics,
- ❑ to see on the non-linear graph actions taking place non-uniformly,
- ❑ to see the relation between a non-linear graph taking place for non-uniformly physical action and a non-linear graph from the field of mathematical functions (function of quadratic type, fractional rational type, power type, exponential type, etc.),
- ❑ to write down a general physical equation,
- ❑ to transform the graph of non-linear function into the graph of linear function,
- ❑ to construct a linear graph of physical dependency (to choose the suitable co-ordinate system),

- to write down a physical equation and to determine values of physical quantities and constants, either as the slope of a straight line (quotient $\Delta y/\Delta x$) or at a point of one of the co-ordinate axes (after graphic extrapolation).

2 A block scheme of examining physical dependence by a graphical method, using a computer

Nowadays many experiments in a physics laboratory are computer-controlled, the output of such experiments mostly being graphs visualising mutual dependencies of physical quantities. Graphic visualising is either of a linear or non-linear nature and students may, directly or after certain mathematical modifications, read varied physical information from them. In examining physical dependencies displayed by a computer, it is necessary to take into account the significance of the sequence of steps. At the department of Physics of the Faculty of Natural Sciences, we have considered the methodology of examining physical dependencies, taken down or displayed by computer. The next part of our paper presents a block scheme of the examination of physical dependency by graphical method by means of computer.

A block scheme [6] leads students, in a mathematical way, in their processing of results of the physical measurement to the expression of physical dependency and to the determining of values of physical quantities and constants by means of computer (Fig. 1). The presented block scheme is generalised for computer processing of physical dependencies.

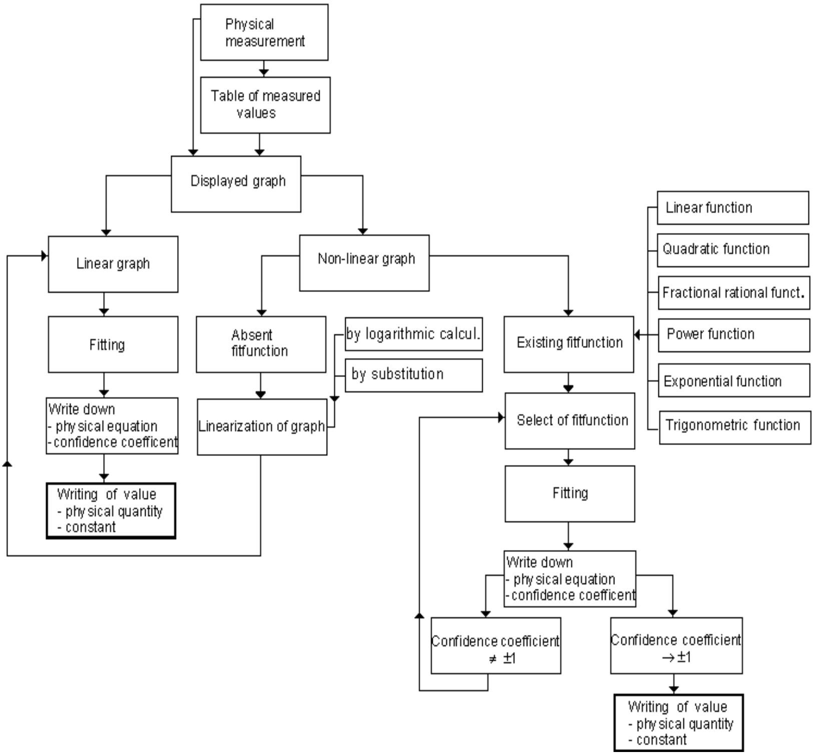

Figure 1

Methodology of examining physical dependency by a graphical method by means of MS Excel

1. Create a suitable MS Excel table from the measured values of physical quantities.

2. Use command Insert/Graph in the right-angled system of co-ordinates to construct a graph of dependency of transistor collector current I_k and the potential $U_{EB.}$

3. For known reasons, it is necessary to fit the visualized physical dependency. Fit the linear as well as non-linear dependency (function of quadratic type, fractional rational type, power type, exponential type, …) in the following way. Right-click on the visualized dependency and use the command Add trend line to choose one of already predefined fitting functions. Choose a fitting function based on the knowledge acquired from the instructions for use of the laboratory task, or from literature in a given field of physics.

4. In the dialogue window select command Options, choose Visualization of regression equation and Visualization of reliability coefficient. Click on OK and the programme will display the fitted dependency, writing out its analytical expression of the regression equation, including the confidence coefficient. If the confidence coefficient value approaches +1 or –1 (for example, 0.996), consider the selection of the fitting function correct.

5. From the displayed regression equation write down the physical equation and, directly from it, read the value of the constant *B*, write what the *B* constant represents and explain how to determine the Boltzmann constant *k* from it, and determine it.

3 Laboratory task solved by a graphical method

Practical verification of Boltzmann law about the division of energy

In the laboratory task, proceed according to the instructions given in the text-book [3] and adjusted for the idea of the graphical method. The dependency of the collector current I_k of the NPN type transistor on the input potential U_{EB} is examined, respecting the relation

$$I_k = I_0 \exp\left[\frac{eU_{EB}}{kT}\right]. \tag{1}$$

The Boltzmann constant is measured at different temperatures, using a Philip Harris measurement system, and the results are processed by means of a graphical method using the MS Excel, based on the methodology. The final value of the Boltzmann constant is compared to the table value.

U_{EB} / V	I_k / mA
0,68	2
0,69	3
0,70	5
0,71	10
0,72	15
0,73	19
0,74	27
0,75	34
0,76	49
0,77	74

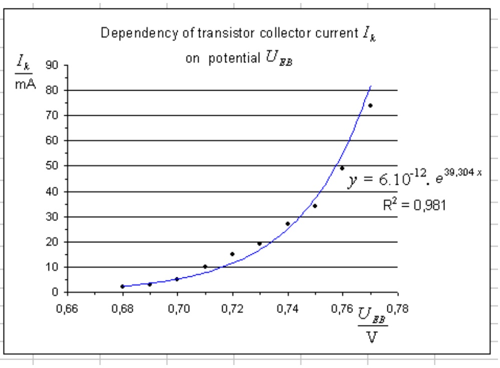

Figure 2

In the physical measurement, the table of measured values is acquired (Fig. 2), and if we proceed in accordance with the block scheme.... and in accordance with a presented methodology ...., we will determine the value of the Boltzmann constant by means of a graphical method.

- ❑ The displayed physical dependency represents a non-linear graph (growing exponential dependency) (Fig 2).
- ❑ Fit this dependency for exponential function and display the regression equation and reliability coefficient.
- ❑ MS Excel fits the dependency and writes out the regression equation in the form of

$$y = A\mathrm{e}^{B.x} = 6.10^{-12}\,\mathrm{e}^{39,304x}\,, \tag{2}$$

and the confidence coefficient R^2 = 0,981. The selection of fitting function may be considered correct if $R^2 \to \pm 1$ (the block scheme and Methodology ...),

- ❑ Expression of growing exponential $y = A.\mathrm{e}^{B.x}$ and physical equation from graph

$$I_k = I_0\mathrm{e}^{BU_{EB}} \Rightarrow I_k = 6.10^{-12}\,\mathrm{e}^{39,304U_{EB}}\,. \tag{3}$$

The laboratory task instructions tell us the constant $B = \dfrac{e}{kT} \Rightarrow k = \dfrac{e}{BT}$, where e is the size of elementary electric charge and T is absolute temperature at which the measurement was carried out. Coefficient $A = 6.10^{-12}$ in regression equation (2) represents in the physical equation (3) the value of current I_0.

❑ Determining Boltzmann constant:

$$k = \frac{e}{BT} = \frac{1{,}602.10^{-19}}{39{,}304.291{,}46} \text{ J}^{\cdot}\text{K}^{-1},$$

k = 1,39845.10^{-23} J·K^{-1}.

A numerical value for Boltzmann constant - k = 1,380662. 10^{-23} J·K^{-1} .

Conclusion

Learning the graphical method as one of the cognitive methods may be evaluated as very positive. A given laboratory task comes from the set of eight laboratory tasks whose results are processed by means of a graphical method, and which is presented to physics teacher-training students within their Physics practical exercises I, II, III and IV at the Department of Physics of the Faculty of Natural Sciences. Examining physical dependencies and processing laboratory measurement results by means of the presented block scheme, as well as the proposed methodology, has proved to be useful. A higher level of knowledge about the graphs of functions with a physical content has manifested itself especially in the form of correctly evaluated protocols of laboratory measurements.

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Unit title	**The special Theory of Relativity and Time Dilation**
Topic	Einstein and relativity concepts such as time, light and gravity
Name and email address of person submitting unit	Not available
Aims of Unit	This unit encourages pupils to think about Einstein's special theory of relativity. It requires pupils to absorb information and relate this to the development of a theory.
Indicative content	Gravity, time, light Relativity and Muons.
Resources needed	The pupils will require access to the paper in either hard or electronic form.
Teachers notes	This is a challenging main task appropriate for pupils who are 16+ the whole task will require at least 50 minutes. This task requires pupils to discuss and digest information and then respond to embedded questions which require calculations. Learning Outcomes All students will be able to discuss some key points of the theory of special relativity. Most students with some help will be able to attempt the embedded tasks within this presentation. Some students unaided will complete the calculations and relate these to observations and theoretical predictions.

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The Special Theory of Relativity and Time Dilation

Albert Einstein proposed the special theory of relativity in 1905. At first the scientific community saw little special about it and not much interest was shown. Consequently the theory revolutionised 20th century physics, and 100 years later Einstein's life and work are well known by the general public worldwide.

Einstein's proposal was based on two ideas:

that the speed of light in vacuum is constant for all observers no matter how they are moving

that the laws of physics are the same for all systems that move relative to each other at a steady speed.

Einstein used *gedanken* (thought) experiments to develop his ideas, and much is written about how he imagined what things would look like if he could ride on a beam of light. But he also built on and acknowledged the work of others, (Maxwell, Lorentz, Poincare to name a few), like any other scientist. The special theory of relativity is often expressed in terms of mathematical equations outside the grasp of most people, yet essentially it is the concept that is most challenging to students of the theory. On this Einstein believed that the thinking involved was most suited to a child, and said,

„The normal adult never bothers his head about space-time problems. Everything there is to be thought about it has already been done in early childhood. I, on the contrary, developed so slowly that I only began to wonder about space and time when I was already grown up."

In Einstein's view therefore the concepts, if not the mathematics, of the special theory of relativity should be accessible to students of all ages. This piece of work, including student activity, tackles one consequence of the special theory, time dilation.

Time dilation means that a process that takes a certain time to occur in a moving system is observed to take a longer time by someone outside the system. Accepting Einstein's advice that this concept can be understood using child-like thinking, let's consider the principle using an analogy from the fairground.

Imagine a Ferris wheel moving clockwise at a known steady, slow speed. A girl watching the wheel and her friends in different cars of it, decides she can tell the time by watching the cars pass a fixed point as long as she knows the start *time and the number of cars that pass the point each hour. She probably isn't concerned that she sees the cars by photons of light reflected from them, and these travel to her at the speed of light – she doesn't need this idea to tell the time! Scientists say the girl is in* a stationary frame of reference *when she observes the wheel like this.*

Now ask yourself what happens if she moves. Let's say she moves in a circle too, inside the wheel but centred at the same point, but she moves slower than the wheel.

The cars still pass her, and she still sees them by reflected photons of light moving at the same speed according to Einstein, but the cars will take longer to pass because she is moving. The time that adjacent cars take to pass her has increased because her frame of reference is no longer stationary, and this change affects her method of telling the time. She may think that time is passing more slowly! This is the idea of time dilation.

The mathematical equation used to describe it, (not too difficult to justify, and to be found in any advanced school physics textbook), is

$$\Delta t' = \varphi \Delta t \ ,$$

where $\Delta t'$ is time measured in a frame of reference moving with velocity v relative to a stationary one in which the time interval is Δt. The quantity φ is called the Lorentz factor – it is a measure of v relative to the speed of light c, and

$$\varphi = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \ .$$

How time dilation explains the paradox concerning muons formed in the atmosphere

Muons are sub-atomic particles sometimes formed in the atmosphere as a result of cosmic ray collisions. They can be detected using balloon flight experiments 2 km above mountain observatories. The paradox is that something like 80 % of the muons detected by the balloons are also detected at the observatories, and a few calculations will show you that is well in excess of what classical mechanics predicts.

ACTIVITY:

Muons travel at 0.996*c*. How long will it take them to travel 2 km? (the speed of light in vacuum, $c = 3.0 \times 10^8\ \mathrm{m \cdot s^{-1}}$)

The muons are unstable and decay with a half-life of 2.2 μs. How many whole half-lives elapse in the time taken for them to travel 2 km?

Given that in one half-life the muon number decreases to one half of its original value, how many of the original muons will remain after three half-lives?

So measurement shows that 80 % of the muons remain to be detected at the observatory – your calculations predict only 12 % remain. Quite a discrepancy! However, travelling at 0.996*c*, special relativity says that the half-life (defined in the muon's moving frame of reference) will be longer if measured at the observatory. A few more calculations will help you to account for the paradox.

ACTIVITY:

Travelling at 0.996*c*, what is the value for the Lorentz factor φ, for the muons?

If the muon's half-life is 2.2μs in its moving frame of reference, what will it be in the observatory?

In the 6.7μs the muon takes to travel the 2 km, how many half-lives have elapsed?

In this time, a fraction of a half-life, only about 20 % of the muons decay solving the paradox!

So time dilation, a consequence of Einstein's theory, perfectly explains experimental measures of muons created in this way.

Once we can appreciate the idea of time dilation it can be used to realise that it leads to length contraction. A physicist moving in a space laboratory with the muons would measure the 6.7 μs travel time recorded in the mountain observatory as 6.7/11.2 μs, about 0.6 μs. She would then measure the distance the muons travel to be shortened from 2 km to 0.6 μs × 0.996*c*, about 180 m. Conversely, a 2 000 m long object moving towards the observatory at this speed would be measured as 180m long.

Looking at the mathematical form of the Lorentz factor, you should be able to see that unless the speed of an object is significant in comparison with the speed of light, these time dilations and length contractions are insignificant explaining why classical mechanics correctly describes the motions of athletes, cars and Ferris wheels (though the latter acts as a useful analogy!) However, observation involves light, meaning photons travelling at the speed of light, and because time and space are interdependent all measurements of them are relative to the measurer's frame of reference.

Einstein's special theory of relativity led on to his famous equation $E = mc^2$ (and therefore nuclear fission and stellar evolution, as well as t-shirts!) and plenty of other developments in 20th century physics. But it also explained phenomena supposedly understood before this time. For instance, the magnetic forces generated when a current flows in a conductor are a consequence of time dilation, and can only be fully understood using this principle.

Unit Title	**How is it to be a teacher – Meteorology**
Topic	Meteorology
Name and email address of person submitting unit	Ľubomíra Valovičová lvalovicova@ukf.sk
Aim of unit	Students try to be a teacher and this can help them better understand the topic of the lesson. This voluntary activity can improve their relationship to physics
Indicative Content	10 lessons, age of students 12–13
Resources needed	internet, data projector, literature
Teachers notes	1. lesson: basic terms, climate and weather. 2. lesson: layers of the atmosphere 3. lesson: condensation of water vapor 4. lesson: humidity of air 5. lesson: cloud and precipitation 6. lesson: wind and its direction 7. lesson: meteorological map 8. lesson: meteorological station 9. lesson: pollution of atmosphere 10. lesson: disasters caused by weather

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	The lesson isn't taught by the teacher but by the students. The teacher divides the themes among the students who prepare the lessons. It is convenient to have two students for one topic so neither speak for longer than 20 minutes. Teachers have to give the students sufficient time for preparation. It is best to give one month before the beginning of the meteorology topic, so the students have enough time to consult with the teacher and improve their performance. In the lesson the teacher sits at the back of the class and watches the lesson. Students may use data projector for presentation.

An introduction lesson has to be done at least one month before the start of the project.

A teacher introduces the topic of each individual lesson.

1^{st} lesson: **An Introduction to Meteorology. Weather and Climate.**

2^{nd} lesson: **Spheres of the Atmosphere.**

3^{rd} lesson: **Air humidity.**

4^{th} lesson: **Liquifaction of Water Vapor.**

5^{th} lesson: **Clouds and precipitation.**

6^{th} lesson: **Alteration of atmospheric air pressure. Wind.**

7^{th} lesson: **Weather station.**

8^{th} lesson: **Weather chart and weather forecast**

9^{th} lesson: **Air pollution.**

10^{th} lesson: **Weather-related catastrophes.**

The teacher may employ **two methods of topic distribution:**

1^{st} method: **Lottery**

Division of students into groups is determined by a lottery system. Two lottery jars are used. One jar contains slips marked with one of the following digits: 1, 2...10 representing the topics of the lessons, the other one contains slips with the students' names written on them. The slips are randomly selected from each jar. The teacher draws lots to decide on groups and the topic. There are two/three slips drawn from the jar containing the names and one slip from the jar containing the slips with topics. The students chosen will work on the pertaining topic as a group. The group size can vary according to a total number of students.

2^{nd} method: **Auction**

The teacher allows the students to create groups on their own. Then the teacher opens an auction on the topics. He/She announces the first topic to be "sold" on auction. The leader of the group interested in “buying” the topic raises his/her hand. When there are more groups interested in the same topic, they may compete for the topic. The teacher asks questions on the facts from previous lessons. The group that answered more questions correctly is the winning one.

Each group obtains a card with the **topic of the lesson** + **a set of questions** that should be given at the end of the lesson to see if the objectives of the lesson are achieved.

Topics:

1st lesson: ***WEATHER AND CLIMATE***

Explain the terms weather and climate. Climatic changes.

2nd lesson: ***SPHERES OF THE ATMOSPHERE***

Define each individual sphere of the atmosphere. (the more interesting and unusual the better)

3rd lesson: ***AIR HUMIDITY***

Definitions of air humidity**.** How can air humidity be measured? What is dew point? What is dew point temperature? Absolute and relative air humidity.

4th lesson: ***LIQUIFACTION OF WATER VAPOR***

The origin of clouds. Cloudiness.

5th lesson: ***CLOUDS AND PRECIPITATION***

Cloud Types. (common cloud classifications). Explain the terms dew and fog. Definitions of precipitation. What causes precipitation. Why do we measure the amount of precipitation? How do we measure precipitation?

6th lesson: ***ALTERATION OF ATMOSPHERIC AIR PRESSURE. WIND.***

What is air pressure? High pressure **(H)** and low pressure **(L)**. What is wind and how does it move? Wind direction and speed.

7th lesson: ***WEATHER STATION.***

What is the purpose of weather stations. Weather Station Inside.

8th lesson: ***WEATHER CHART AND WEATHER FORECAST***

Weather Charts: understanding the charts. Weather Forecast. Warm and Cold Fronts.

9th lesson: ***AIR POLLUTION***

Classification of air pollution sources, natural and artificial sources of air pollution.

Greenhouse effect and ozone layer.

10th lesson: ***WEATHER-RELATED CATASTROPHES***

Give details on natural disasters, e.g. hurricanes, tornadoes, snow and ice storms, floods and droughts.

After all the topics have been chosen, the teacher will display a collection of links to the most appropriate web pages in order to facilitate student's search for the relevant information. This is also beneficial for the teacher, as he/she can more easily track and evaluate students' work. It gives the teacher information about the students' abilities to grasp information from the Web using more than one source. The content of the presentations should come from many different web pages, and not only from a single one.

Students learn to use the internet as a learning and teaching resource.

At the very end, if there is some time left, the teacher invites the students to ask whatever questions they have.

Note: Do reserve enough time for the activity.

Time for consultation:

The work on the presentation lasts a few weeks, and is conducted in form of the students' consultation with the teacher. The teacher makes himself/herself accessible to groups. At least a small amount of scheduled class time should be reserved to discuss the issues and provide students with support and advice related to the topic. Students have to prepare and present lesson plans to the teacher. The teacher provides advice to students on how to conduct a review of material from the lesson (e.g. preparing of review questions asked at random, preparing of crosswords, riddles etc.). The students-teachers should prepare a brief summary containing the most important facts from the topic presented. This summary is to be distributed among the students at the end of the lesson.

Students' lessons:

As the teacher takes the role of a student, it is very important for the teacher to fully identify himself/herself with this role. The teacher takes the seat at a back-row desk and monitors the lesson. The position of the teacher contributes to a more relaxed atmosphere in the class. The teacher puts down the notes, answers the students' questions, and eventually acts as an interrupting element in the class (the best is to behave completely the same way as the student during the teacher's lesson. If the student misbehaves and interrupts the teacher, the teacher does the same things to the student-teacher during his/her lesson.). This is the most effective and efficient method to make the student to think about his/her own behavior during the lesson.

The students-teachers gain experience presenting in front of the class. Many of them may lack confidence or feel tense. The teacher should try to ensure that these students do not feel "put down" by their extrovert "colleagues". There is no need for the teacher to interrupt the students' presentation even if there is a slight chaos and noise in the class. If the teacher does interrupt the students' idea of the lesson, students might get lost and feel insecure. The students should prepare the lesson in order to make the others to be interested in the topic presented.

The teacher takes notes in order to record the facts being said and presented by students.
The aim of note taking is not to focus on student's weaknesses**,** but rather to ascertain their strengths.

The most significant role of the teacher is to evaluate group work contribution and presentation performance.

The teacher evaluates:

- *Overall topic presentation*
- *Extra work* (crosswords, riddles, jokes ...)
- *Teaching techniques* (e.g. instructing, explaining, demonstrating, questioning, illustrating etc.) *and aids* (e.g. using visuals, experimenting etc.) *used*
- *Distribution of a brief summary*
- *Discipline in the class*
- *Lesson planning and group co-operation*

At the end of the thematic unit, the students should write a short review test from the material presented. The teacher may provide the students with a brief summary of learning material if necessary.

METEOROLOGY – summary

WEATHER – is connected with processes in the atmosphere. It is a state of the atmosphere at a certain time and place. There are six basic parameters of the weather: *the air temperature, air pressure, air flow/movement (wind), air humidity, cloudiness, and rainfall/precipitations.*

CLIMATE – the state of the atmosphere at a certain place in the long run. It is caused by the climate and geographic factors. The main attributes of the climate are the air temperature, precipitations, and winds. The Earth climate is very diverse. There are five types of the climate. Our climate is mild.

ATMOSPHERE – the gaseous envelope of the Earth. It is a necessary precondition for life. According to physical properties, it is divided into five spheres:

1. troposphere – up to about 7.5 mi above the Earth's surface. It is the thermal layer of the atmosphere because it is warmed by the sunshine reflected from the Earth's surface. The temperature decreases as the altitude increases. The overwhelming majority of air and all of the water vapor is in this sphere. All the processes generating the weather take place here as well.

2. stratosphere – up to about 30 mi. It contains very small amount of water vapor so that no clouds are formed here. Ozone is concentrated in this sphere in the ozone layer - it is a protecting shield of the Earth. Ozone absorbs a large amount of the ultraviolet part of the Sun's radiation. The temperature is higher than in the troposphere.

3. mesosphere – from 30 to 45 mi. The coldest part of the atmosphere. It is so cold that clouds of ice are formed. They are visible only at night when the setting Sun is illuminated from below.

***4. thermosphere* –** up to about 60 mi, the last layer before space. Aurora borealis (northern lights) appears here. Meteorites can also be observed in this sphere. It is divided into two parts: ionosphere and magnetosphere. Thermosphere has the smallest air density.

***5. exosphere* –** the highest and least dense layer. The atmospheric pressure gradually decreases to zero and the Earth's atmosphere turns into interplanetary space.

AIR HUMIDITY – determined by the amount of water vapor in a given volume. Humidity decreases with increasing altitude. It can be absolute or relative. *The absolute air humidity*: water vapor mass contained in 1 m^3 of air. *The relative air humidity*: the ratio of the absolute air humidity and the largest absolute air humidity at a given temperature. The unit is per cent. The upper level of relative humidity occurs in misty weather. The humidity level is important to monitor when storing fruit, food, furniture, etc.

WATER VAPOR – present in the atmosphere through water evaporation from the ground, water areas, animals and plants. Water perpetually circulates between the Earth and atmosphere.

DRY AIR – contains almost no water vapor. The driest air is over the subtropical deserts.

SATURATED AIR – contains a large amount of water vapor. When the amount of water vapor is increased tiny drops (steam, clouds, mist) appear. The most saturated air is over the equatorial areas and, especially, over the oceans.

DEW POINT – the temperature at which water vapor is liquidized in the air.

HUMIDITY METER (HYGROMETER) – device for measuring the humidity (the scale is in %). *Hair hygrometer* is most often used – it consists of a bundle of stretched hairs, arrow, and a scale. When the air is dry (moist), the hairs become shorter (longer).

CLOUD – a large amount of water droplets or icy crystals observed in the sky.

CLOUDINESS – the amount of clouds over a certain place. It is one of the basic meteorological measures. It is determined by estimation. The graduation used in weather forecasts is: sunny, mostly sunny, partly cloudy, mostly cloudy, and cloudy.

DEW – the most cooled objects during the night are thin objects like blades, leaves, etc. Vapor in the air liquidize on these object, thus creating dew. When the air temperature is below 0ºC, frost occurs.

CLOUDS – most easily determined by the shape and the altitude. According to the altitude, clouds are divided into three groups in which there are ten types of clouds. The first group – *high level clouds* – are composed of ice crystals. The second group – *medium level clouds* – are composed of ice crystals and water droplets. The third group – *low level clouds* – are usually composed of water droplets. It is rare to observe a cloud fitting into exactly one group. According to the shape, there are three groups of clouds: *Layer clouds* – white or gray cover of the sky. In smaller altitudes they create thick layers and bring about rain or snow. In higher altitudes they have the shape of gray or white veils. At the altitude of about 3 mi they form white flakes. *Cumulous clouds* – large clouds with distinct edges. On the underside they are mostly flat, on the top they are round. Illuminated parts are glaring, the parts in the shadow are gray. In the summer period they are connected with storms, but they can appear in the sky also during nice weather.

MIST(FOG) – a cloud near the Earth's surface. It appears when the air near the ground has a large relative humidity and is suddenly cooled.

AIR PRECIPITATIONS – a cloud consists of a huge number of water droplets or ice crystals. If they are small, even weak air currents may transfer them upwards. Droplets or crystals gradually cluster in a cloud, thus increasing their volume and mass. Dew and mist are examples of air precipitations.

PRECIPITATION AMOUNT – total amount of water that falls down at a certain place within a certain time interval. It is measured in inches. Hence, the amount of water corresponding to measure 1 over the area of 1 sq. foot is 144 cubic inches or 0.623 gal.

PRECIPITATION METER – a cylindrical container with a funnel of a given area in the top part of the container. Precipitated water is stored in a container at the bottom of the precipitation meter.

ATMOSPHERIC AIR PRESSURE – measure of the air pressure at a certain time over a certain place.

ISOBARS – a curve connecting places of constant air pressure at a certain time.

LOW (L) – an area of low air pressure.

HIGH (H) – an area of high air pressure.

BAROGRAPH – a device for continuous measurement and recording of air pressure values.

WIND – air movement caused by pressure differences in atmosphere.

WIND DIRECTION – indicated by the cardinal points. It is also influenced by the Earth's rotation.

WIND SPEED – depends on the value of the air pressure difference between places. It is measured in mph. It is not steady, but changes often. An average speed is thus indicated.

ANEMOMETER - used to measure the direction and speed of wind.

METEOROLOGICAL STATION – serves for weather observation. Weather is measured there according to international standards and units. A station is situated in a garden with a fence and far away from houses and buildings. It consists of:

Meteorological cell – thermometer, hygrometer, and pressure meter.

Grassless area – used to detect the state of soil.

Thermometer in grass – detects the lowest ground temperature.

Anemometer.

Heliograph – records the length of sunrise.

Balloon (wind direction and speed measurement) *and theodolite* – device for determining the balloon position.

Precipitation meter (*ombrograph* – automatic recording of precipitation amount).

WEATHER MAP – records information on weather. Atmospheric fronts are of great importance. Other information is wind direction, precipitation and storm occurrence, atmosphere state, etc.

WARM FRONT – a zone of contact of warm and cold air that shifts to the side of cold air. After its withdrawal a given place is warmed.

COLD FRONT – the opposite of warm front.

WEATHER FORECAST – designed for the public. It contains the information on precipitations, extreme day and night temperatures, the forecast of the wind directions and speed. Special forecasts are available for power supply engineers, farmers, transportation, areas of active volcanoes or tectonic zones.

AIR POLLUTION – natural sources of pollution – volcanoes and zones of volcanic activity, dust storms, and putrefactive processes. Artificial pollution – industry (carbon dioxide, radioactive fallout, freons damaging the ozone layer). Carbon dioxide in the atmosphere transmits the Sun‘s rays but absorbs radiation from the Earth’s surface. As a consequence, air temperature increases – the greenhouse effect.

Illustration annotation:

CLOUDS

They are created from rising air. When air rises upwards, it is cooled down and a part of the water vapor condenses into droplets or freezes and forms small ice crystals.

Sorts of clouds:

1. Cell clouds – composed of small ice crystals (3–7.5 mi).
2. Cumulous clouds – separated cloud masses (medium altitude).
3. Cirrus clouds – formed in high altitudes, contain only ice crystals.
4. Stratus clouds – heavy and stretched out across the sky like a blanket. Gray stratus precedes rain.
5. Cumulonimbus – a type of cumulous cloud that brings storms (high altitudes).

MIST (FOG) – It is a cloud lying near the ground. It is formed when air near the ground has a large relative humidity and is rapidly cooled. It is most often formed over wet lands (spring and fall fogs), water surfaces, and near rivers in the evenings or at night. In the morning it lasts only till the Sun sufficiently warms the ground at a given place.

TYPES OF CLOUDS

1. the Sun and the Moon shine through them like through greasy/oily glass.
2. brings tornados and storms.
3. brings snow and rain.
4. highest situated clouds having the form of fibres.

The amount of clouds in the sky over a certain place is called cloudiness. There is a five-grade classification of cloudiness: sunny, mostly sunny, partly cloudy, mostly cloudy, and cloudy

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