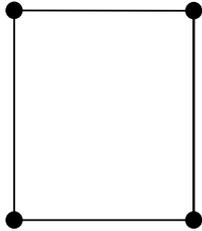


Unit title	Graph Theory and the Bridges of Königsberg
Topic	Graph Theory
Name and email address of person submitting unit	Christine.Farnsworth@sunderland.ac.uk
Aims of Unit	To explore the conditions for drawing a Eulerian graph and applying the findings to the 'Königsberg bridges problem'.
Indicative content	Eulerian, semi-Eulerian and non-Eulerian graphs.
Resources needed	Paper and pencils.
Teachers notes	This activity introduces historical aspects of mathematics. Euler considered the 'Konigsberg bridges problem' in more generality and published what can be thought of as the first research paper in graph theory.

The Bridges of Königsberg.

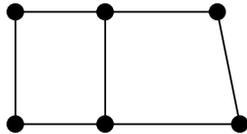
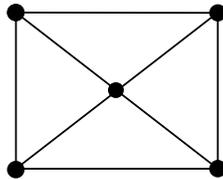
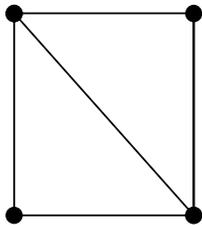
Graphs are diagrams which join points (vertices) with lines (edges). The degree of each vertex is defined as the number of edges coming out of the vertex.

The graph below has 4 edges and 4 vertices and each vertex has degree 2.



For each of the graphs below find:

- the number of edges;
- the degree of each vertex;
- the total of the degrees.



Is there a connection between the number of edges of a graph and the total of the degrees of the vertices? Examine some graphs of your own.

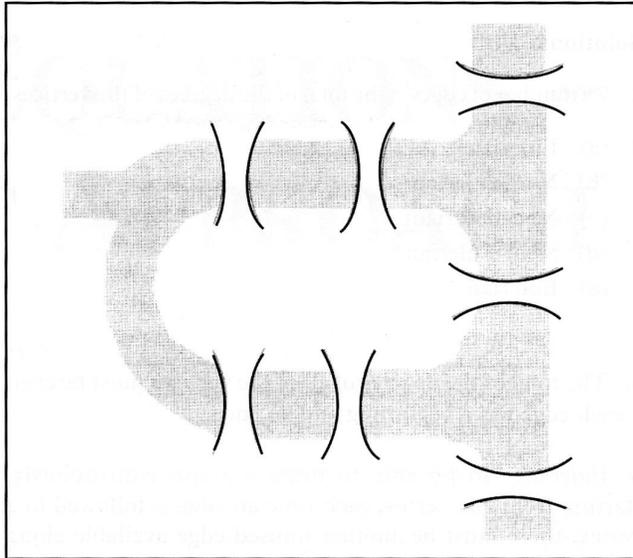
Is it possible to draw these graphs continuously i.e. without lifting your pencil from the paper and without retracing any edge?

A graph that can be drawn continuously starting at any vertex and finishing at the starting point is known as Eulerian. If the graph can be drawn continuously

but only starting from certain vertices it is known as semi- Eulerian and if it cannot be drawn continuously it is non-Eulerian.

Try to establish and explain the conditions for each type of graph.

Consider the Königsberg bridge problem. Is it possible to walk over each of the seven bridges only once?



Seven bridges in Königsberg are arranged as shown.

(Ref. Mathematics Association (2001) Above and Beyond Graph Theory. *Mathematics in School*. 30 (3) pp. 12-13)