

Unit title	Estimation of π
Topic	Shape and Space
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Aims of Unit	To consider a historical approach to the estimation of pi using Archimedes' Method.
Indicative content	Students will find the perimeter of an inscribed regular hexagon ($3d$ where d is the diameter of the circle) and the perimeter of a circumscribed regular hexagon ($2\sqrt{3}d$). Using a spreadsheet they will then extend the activity to investigate perimeters of polygons with 12, 24, 48, 96, n sides and so obtain an estimation of the circumference of the circle and π .
Resources needed	Paper, pencils, protractors, compasses, computer spreadsheet.
Teachers notes	Students should have pre-requisite knowledge of trigonometrically relationships in right-angled triangles. Materials on web site

Estimation of π

The Greek symbol π was first used to denote the ratio of the circumference of a circle to its diameter in 1706, by the Englishman William Jones. The same symbol had previously been used for just the circumference.

Archimedes (c.250 BC) was a Greek mathematician who estimated the value of π by calculating the perimeters of inscribed and circumscribed regular

polygons. He found that $\frac{223}{71} < \pi < \frac{22}{7}$ where both limits give an

approximation of π as 3.14 correct to 2 decimal places. Archimedes based his method on the observation that if a circle is enclosed between two polygons of n sides, then as n increases, the gap between the circumference of the circle and the perimeters of the inscribed and circumscribed polygons diminishes, so that eventually the perimeters of the polygons and the circle would become identical.

(G. G. Joseph (1991) *The Crest of the Peacock*. London: Penguin)

Find the perimeter of an inscribed regular hexagon where the radius of the circle is r cm. Express the perimeter in terms of the diameter of the circle. Also find the perimeter of a circumscribed regular hexagon where the radius of the circle is r cm. Again, express the perimeter in terms of the diameter of the circle.

Use a spreadsheet to investigate the perimeters of inscribed and circumscribed regular polygons with 12, 24, 48, 96,192 sides. What approximation can be given for π ?